THE USE OF EXTERNALLY BONDED CFRP SHEETS FOR SHEAR STRENGTHENING OF I-SHAPED PRESTRESSED CONCRETE BRIDGE GIRDERS

by

ROBIN L. HUTCHINSON

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Structural Engineering Division Department of Civil and Geological Engineering University of Manitoba Winnipeg, Manitoba

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ABSTRACT

The repair and rehabilitation of civil engineering structures is a rapidly expanding industry. Worldwide, structures that have deteriorated with exposure and time are being subjected to increasing modern load levels. The demand for increasingly heavier truck loads is forcing bridge owners to upgrade existing structures.

The City of Winnipeg, Manitoba, Canada is considering upgrading the Maryland Bridge using CFRP sheets, since analysis conducted using current codes indicates that the shear strength of the bridge girders is not sufficient to withstand increased modern truck loads. The Ministry of Transportation of British Columbia is also considering the use of CFRP sheets to increase the shear capacity of the John Hart Bridge in Prince George, B.C. Like the Maryland Bridge, the John Hart Bridge consists of I-shaped prestressed concrete AASHTO girders, which are deficient in shear under the new truck loads.

The use of externally bonded Fibre Reinforced Polymer (FRP) sheets provides an excellent solution for the repair and rehabilitation of civil engineering structures. Since the strength to weight ratio of FRP materials is extremely high in comparison with traditional materials, the installation of continuous light-weight FRP sheets or strips is
remarkably simple in comparison with conventional strengthening techniques. The high tensile strength of FRP materials allows for the use of very thin sheets, which can easily be manipulated to bond to complex shapes and cross sections. FRP materials are not subject to electrochemical corrosion, providing for a durable strengthening solution.

While the use of FRP sheets or strips for flexural strengthening of concrete structures has been studied extensively, the study of externally bonded FRP sheets for shear strengthening has been limited. Due to a lack of information on the use of CFRP sheets for shear strengthening of I-shaped prestressed concrete AASHTO girders, an experimental program has been undertaken at the University of Manitoba, to test scaled models of I-shaped concrete bridge girders strengthened with CFRP sheets.

Seven prestressed concrete beams were strengthened using three different types of CFRP sheets and ten different CFRP sheet configurations. The beams were tested to failure at each end to determine the most efficient strengthening scheme. The contribution of the CFRP sheets to the enhanced shear capacity of the girder is examined, with emphasis on the effect of this particular girder shape. Since the bond between the CFRP sheets and the concrete is a critical component of this strengthening technique, a series of fifteen bond specimens was tested in order to determine the bond characteristics.

Design guidelines and recommendations for the use of externally bonded CFRP sheets for shear strengthening of I-shaped prestressed concrete girders are developed, and are intended to contribute to the current state-of-the-art and the development of design codes.
A rational model is introduced, to predict the shear resistance provided by FRP sheets externally bonded to I-shaped concrete girders. For I-shaped sections, the concrete substrate is subjected to both peeling and shear stresses and failure is initiated by straightening of the CFRP sheets. The shear resistance provided by the FRP sheets is calculated based on the maximum strain developed in FRP sheets and the FRP strain distribution model introduced for I-shaped sections.

Depending upon the configuration of the FRP sheets and the corresponding mode of failure, the internal steel stirrups may not reach yield prior to the initiation of failure. The stirrup contribution to shear resistance is therefore based on the load sharing relationship between the FRP sheets and the steel stirrups, and the predicted mode of failure.

The versatility of the proposed rational model is demonstrated, by applying the model to the traditional ACI approach to shear design, the modified sectional-truss model, the compression field theory and the modified compression field theory. The shear capacity of each beam tested in this experimental program is predicted using the shear prediction models listed above in combination with the proposed rational model, and the predictions are compared with the test results. It is shown that the most accurate shear resistance prediction is obtained using the ACI approach, but with a more realistic prediction for the shear crack angle based on the angle of principal compressive stresses at cracking.
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LIST OF SYMBOLS

a  maximum aggregate size (mm)
A_r  area of FRP sheets (mm^2)
A_s  area of steel (mm^2)
A_{sv}  cross sectional area of steel bars per stirrup (mm^2)
b_w  minimum width of web (mm)
d  distance from centroid of flexural reinforcement to extreme compression fibre
d_v  distance between tension and compression chords in truss model (mm)
d_r  depth of cross section over which FRP sheets are effective (mm)
E_r  modulus of elasticity of FRP sheets (MPa or GPa as noted)
E_s  modulus of elasticity of steel bars (MPa or GPa as noted)
f_t  tensile stress in diagonally cracked concrete (MPa)
f_2  compressive stress in inclined concrete strut (MPa)
f_{2,\text{max}}  maximum compressive stress in diagonally cracked concrete (MPa)
f_{b,\text{bend}}  bond strength obtained from the bending type bond test (MPa)
f_{b,\text{shear}}  bond strength obtained from the shear type bond test (MPa)
f_{b,\text{tens}}  bond strength obtained from the tension type bond test (MPa)
f_{c'}  concrete compressive strength (MPa)
f_{pc}  compressive stress in concrete at centroid of cross section (MPa)
f_{se}  effective stress in steel stirrups (MPa)
f_{sv}  stress in steel stirrups (MPa)
f_y  yield strength of steel bars (MPa)
h  overall height of concrete cross section (mm)
K  constant for determining concrete contribution for modified sectional truss model
L  centre to centre distance between cracks (mm)
L_e  effective bond length of FRP sheets (mm)
$M_{cr}$ moment causing flexural cracking (Nmm)
$M_{\text{max}}$ maximum moment at cross section due to factored loads (Nmm)
$n_f$ number of FRP sheets per one side of beam
$N_v$ Axial force due to shear only (N)
$P_{f_{\text{max}}}$ maximum force developed in FRP sheets (N)
s centre to centre spacing between steel stirrups along longitudinal beam axis (mm)
$s_{cr}$ crack spacing (mm)
$s_r$ centre to centre spacing between FRP sheets along longitudinal beam axis (mm)
$t_f$ thickness of FRP sheets (mm)
$v_{cu}$ shear strength of the concrete substrate (MPa)
$v_{ci}$ shear stress at crack interface (MPa)
$V$ applied shear force (N or kN as noted)
$V_c$ concrete contribution to shear resistance (N)
$V_{ci}$ concrete contribution to shear resistance based on flexural-shear cracks (N)
$V_{cw}$ concrete contribution to shear resistance based on web shear cracks (N)
$V_d$ shear force at cross section due to unfactored dead load (N)
$V_f$ FRP sheet contribution to shear resistance (N)
$V_{f_{\text{max}}}$ FRP sheet contribution to shear resistance at onset of sheet straightening (N)
$V_i$ maximum shear occurring simultaneously with $M_{\text{max}}$, due to factored loads (N)
$V_n$ nominal shear resistance of beam (N)
$V_p$ vertical component of draped prestressing strands (N)
$V_s$ steel stirrup contribution to shear resistance (N)
$V_{se}$ steel stirrup contribution to shear resistance based on effective stirrup strain (N)
w crack width (mm)
$w_f$ width of FRP sheet perpendicular to principal fibre direction (mm)
z lever arm between tension and compression chords in truss model (mm)
$\alpha$ angle of inclined steel stirrups (degrees)
$\alpha_r$ angle of principal fibres of FRP sheets (degrees)
$\beta$ constant for determining concrete contribution, modified compression field theory
$\varepsilon_t$ principal tensile strain in web
\( \varepsilon_2 \) principal compressive strain in web

\( \varepsilon_c \) strain in concrete

\( \varepsilon_{c'} \) strain in concrete at \( f_c' \)

\( \varepsilon_{cr} \) cracking strain in concrete

\( \varepsilon_r \) strain in FRP sheets

\( \varepsilon_{f\text{ave}} \) average strain in the FRP sheets

\( \varepsilon_{f\text{max}} \) maximum strain developed in the FRP sheets

\( \varepsilon_s \) strain in steel stirrups

\( \varepsilon_{se} \) effective strain in steel stirrups

\( \varepsilon_{s\text{max}} \) maximum strain in steel stirrups

\( \varepsilon_{x, x'\text{t}} \) total strain in longitudinal direction in web

\( \varepsilon_{xm} \) strain in longitudinal direction in web due to applied moment

\( \varepsilon_{xp} \) strain in longitudinal direction in web due to prestressing

\( \varepsilon_{xv} \) strain in longitudinal direction in web due to shear only

\( \varepsilon_y \) strain in transverse (vertical) direction in web

\( \gamma_{fs} \) ratio of vertical component of strain in FRP sheets to strain in steel stirrups

\( \rho_{\text{fp}} \) area fraction of FRP sheets

\( \tau_{\text{ave}} \) average bond stress (MPa)

\( \tau_{\text{ult}} \) ultimate bond stress (MPa)

\( \theta \) angle of inclined compressive struts (degrees)

\( \theta_{cr} \) angle of inclined shear cracks (degrees)
INTRODUCTION

1.1 GENERAL

The use of externally bonded Fibre Reinforced Polymer (FRP) sheets provides an excellent solution for the repair and rehabilitation of civil engineering structures. FRP sheets are light-weight with a high tensile strength, and are not subject to electrochemical corrosion. Since the high-strength FRP sheets are extremely thin, they are easy to install, can be applied to complex shapes and cross sections, and facilitate a significant reduction in construction time. When compared to conventional repair and strengthening methods, externally bonded FRP sheets provide rehabilitation solutions that are both cost effective and durable.

Throughout the world, infrastructure that has deteriorated with exposure and time is being subjected to increased modern load levels. The demand for using increasingly heavier truck loads is forcing bridge owners to upgrade existing structures. The use of Carbon Fibre Reinforced Polymer (CFRP) sheets minimizes traffic interruption and provides an effective low-cost rehabilitation solution.
The City of Winnipeg, Manitoba, Canada is considering upgrading the Maryland Bridge using CFRP sheets, since analysis conducted using current codes indicates that the shear strength of the bridge girders is not sufficient to withstand increased modern truck loads. The Ministry of Transportation of British Columbia is also considering the use of CFRP sheets to increase the shear capacity of the John Hart Bridge in Prince George, B.C. Like the Maryland Bridge, the John Hart Bridge consists of I-shaped prestressed concrete AASHTO girders, which are deficient in shear under the new truck loads.

Due to a lack of information on the use of CFRP sheets for shear strengthening of I-shaped prestressed concrete AASHTO girders, an experimental program has been undertaken at the University of Manitoba, to test scale models of the Maryland Bridge girders strengthened with CFRP sheets. Seven prestressed concrete beams were tested to failure at each end to determine the most efficient strengthening scheme. The contribution of the CFRP sheets to the enhanced shear capacity of the girder has been examined, with emphasis on the effect of this particular girder shape. Since the bond between the CFRP sheets and the concrete is a critical component of this strengthening technique, a series of bond specimens have been tested in order to determine the bond characteristics.
1.2 OBJECTIVES

The objective of this research program was to develop design guidelines and recommendations for the use of externally bonded CFRP sheets for shear strengthening of I-shaped prestressed concrete girders. These design guidelines and recommendations are intended to contribute to the current state-of-the-art and the development of design codes. The experimental program was also designed to investigate a specific shear strengthening solution for the Maryland Bridge. Design recommendations for the Maryland Bridge will contribute to a successful field application of this strengthening technique.

The specific objectives of this investigation were:

1. To evaluate the efficiency of the various types and configurations of CFRP sheets tested, and examine the shear resisting mechanisms provided by the CFRP sheets.

2. To investigate the bond behaviour of the CFRP sheets, and determine the specific bond properties of the CFRP sheets.

3. To determine the effect of the concrete cross section configuration on the bond performance and shear resisting mechanisms of the CFRP sheets.
4. To establish load sharing relationships between the internal shear reinforcement and the externally bonded CFRP sheets.

5. To introduce a rational model for the design of shear strengthening schemes using CFRP sheets for I-shaped prestressed concrete girders.

1.3 SCOPE AND CONTENTS

This study consists of an experimental investigation and analytical modeling of I-shaped prestressed concrete girders strengthened with externally bonded CFRP sheets. Based on the results of beam tests, bond tests and the analysis, a rational model is proposed to predict the shear capacity of strengthened prestressed concrete girders. The following is a brief discussion of each phase of the study:

Experimental Investigation – Seven ten metre long prestressed concrete girders were strengthened using three different types of CFRP sheets for ten different sheet configurations. The beams were tested to failure at each end, to examine their behaviour and determine the most efficient strengthening scheme. The test beams were 1:3.5 scaled models of the I-shaped Maryland Bridge girders. Four of the test beams were reinforced using bent-legged stirrups with a shape identical to those used in the Maryland Bridge girders. In order to extend the applicability of the experimental results, the remaining three beams were reinforced with the more commonly used straight-legged stirrup shape.
Six Rectangular tension-type bond specimens were tested, as well as nine Single-Flanged tension-type bond specimens designed to simulate the bottom tension flange of an I-shaped AASHTO bridge girder. The bond properties of the CFRP sheets and the load sharing relationship between the CFRP sheets and the internal steel bars were examined.

**Analytical Study** -- This component of the study includes a rational approach to predict the behaviour and shear capacity of I-shaped prestressed concrete girders strengthened with externally bonded CFRP sheets. The maximum stress that can be developed in an externally bonded CFRP sheet is predicted as a function of the CFRP sheet stiffness based on the effective bond length approach. A bond strength prediction model introduced in the literature is evaluated using the results of the rectangular bond specimen tests. Based on beam test results, the bond strength prediction model is modified to account for the effect of the I-shaped concrete section. A strain distribution model is introduced for CFRP sheets applied to I-shaped cross sections. The load sharing relationship between the externally bonded CFRP sheets and the internal steel stirrups is evaluated using both the bond test results and the beam test results, and a value for the load sharing ratio is proposed. The proposed rational model is applied to different shear strength prediction models including: the traditional American Concrete Institute (ACI) approach, the variable angle truss model approach, the compression field theory, and the modified compression field theory. The reliability of the proposed model, applied to each of the shear prediction models listed above, is evaluated.
**Design Guidelines** -- Based on the results of the experimental investigation and the analytical study, design guidelines are proposed for the use of CFRP sheets for shear strengthening of I-shaped prestressed concrete girders. The design guidelines and proposed rational model can be applied to several existing shear design approaches, as demonstrated in the analytical study.

An overview of the contents of the thesis is provided in the following:

**Chapter 2** – The properties of externally bonded FRP sheets and their application and use for strengthening of concrete structures in general are described in Chapter 2. The bond between the FRP sheets and the concrete is critical for the success of most of these strengthening techniques, and is discussed in detail in this chapter. Two models available in the literature for bond strength prediction are also described.

**Chapter 3** – This chapter is focused on the shear behaviour of reinforced concrete beams. Three recently developed models for predicting the shear capacity of concrete beams strengthened with CFRP sheets are described, although none consider the effect of the I-shaped cross section. Some background on models for predicting the shear capacity of beams without FRP sheets is also provided, as a basis for the models which do consider the contribution of FRP sheets.

**Chapter 4** – Details of the experimental program, which includes fourteen beam tests and fifteen bond tests, are provided in Chapter 4.
**Chapter 5** – Test results for both the beam tests and the bond tests are presented in this chapter. The general behaviour and mode of failure during testing is described, and the test results are analyzed. The maximum strain developed in the CFRP sheets, the strain distribution in the CFRP sheets, and the load sharing relationship between the sheets and the steel stirrups are evaluated and discussed.

**Chapter 6** – In Chapter 6, the contribution to shear resistance provided by the CFRP sheets, the steel stirrups, and the concrete is determined experimentally for each test beam. This analysis forms the basis for the proposed rational model. The proposed model is applied to two existing shear strength prediction models and its reliability is assessed.

**Chapter 7** – The proposed rational model is applied to the compression field theory and the modified compression field theory in Chapter 7. The versatility of the proposed model is demonstrated and its reliability when used in conjunction with the compression field theories is evaluated.

**Chapter 8** – This chapter summarizes the research program, and provides conclusions resulting from this study. Design recommendations and recommendations for further study are also included in this final chapter.
EXTERNALLY BONDED FRP SHEETS FOR STRENGTHENING OF CONCRETE STRUCTURES

2.1 GENERAL

The repair and rehabilitation of civil engineering structures is a rapidly expanding industry. Worldwide, structures that have deteriorated with exposure and time are being subjected to increasing modern load levels. In addition, the continued development of design codes has revealed insufficient safety margins for some structures. As rehabilitation is repeatedly found to be more economical than replacement, existing structures are being strengthened to extend their useful service life as well as to accommodate new uses and functions.

Conventional methods for strengthening of concrete structures such as enlargement of the cross section; external post-tensioning; and externally bonded steel plates, have limitations which must be considered. For example, the additional weight imposed on a structure by section enlargement is significant and may create additional problems in
other structural elements. Weight is also a problem when strengthening with bonded steel plates, and the size of the steel plate must be restricted for handling purposes during installation. Since the heavy steel plates must be applied in pieces of manageable size and weight, construction joints and splicing are required to transfer forces and maintain continuity between the separate lengths of steel plate. Corrosion is another concern with the application of bonded steel plates since the bond at the steel plate-adhesive interface may be compromised and the cross sectional area of the steel may be reduced.

The use of externally bonded Fibre Reinforced Polymer (FRP) sheets and strips is an attractive strengthening alternative in light of the disadvantages of conventional strengthening methods described above. Since the strength to weight ratio of FRP materials is extremely high in comparison with traditional materials, the installation of continuous light-weight FRP sheets or strips is remarkably simple in comparison with conventional strengthening techniques. The high tensile strength of FRP materials allows for the use of very thin sheets, which can easily be manipulated to bond to complex shapes and cross sections. FRP materials are not subject to electrochemical corrosion, alleviating the concerns associated with the use of steel plates, and providing for a durable strengthening solution.

This chapter describes the properties of externally bonded FRP sheets and their application and use for strengthening of concrete structures. The importance of good bond between the FRP sheet and the concrete is presented in this context, and factors affecting the bond performance are discussed. The need for a general model to predict
the bond strength and facilitate the design of concrete structures strengthened with FRP sheets is emphasized, and two empirical models that have been proposed for the prediction of bond strength are discussed.

2.1.1 General Characteristics of Externally Bonded FRP Systems

The strength and stiffness of Fibre Reinforced Polymer (FRP) materials are provided mainly by the high strength fibres. The fibres are embedded in a polymer matrix with a much lower strength and stiffness. Rather than carrying a significant proportion of the applied load, the polymer matrix transfers the load to the fibres through shear stresses at the fibre-matrix interface. The polymer matrix binds the fibres together, supporting and protecting them from premature breakage due to abrasion. The performance of the composite material is greater than the individual components alone, since any broken fibres can continue contributing to the load carrying capacity by load-sharing with other fibres through shear transfer in the polymer matrix.

The three main types of fibres used for civil engineering applications are carbon, glass and aramid. Although short randomly directed fibres have been used in some FRP repair techniques, the fibres typically used for structural applications are continuous in one direction or in two orthogonal directions. The fibres have extremely small diameters, in the range of 10 to 15 μm. For externally bonded FRP applications, the fibres are combined together in very thin continuous sheets, fabrics or strips, with a thickness in the range of 0.1 to 1.6 mm.
In the direction of the fibres, the material properties are controlled by the high strength and stiffness of the fibres and the FRP material is linearly elastic until failure. The strength of a unidirectional FRP sheet in the direction perpendicular to the fibres, depends mainly on the matrix rather than the fibres, and therefore the sheet is extremely weak in this direction. Since the fibres and the polymer matrix are bonded together at the interface, they will undergo the same strain when loaded in the direction of the fibres. In order to utilize the full strength of the fibres, a polymer matrix that has a rupture strain greater than the rupture strain of the fibres is typically used.

One of the main advantages of externally bonded FRP strengthening systems is that they can be tailored to a specific application. A wide range of materials and systems is available and can be categorized into two main types of strengthening systems, FRP sheets and FRP strips. For FRP sheet systems, dry fibre sheets are delivered to the site in continuous rolls, and the polymer matrix is applied on site using a wet-lay-up technique. The polymer matrix typically consists of an epoxy resin that is cross-linked, or polymerized, by the use of a hardener designed for room temperature curing. By comparison, the continuous fibres of the FRP strips are pre-impregnated with the polymer matrix by the manufacturer and the strips are delivered to the site as a composite material. Adhesives are supplied by the FRP strip manufacturer for application of the strips onto the concrete structure.
2.1.2 Strengthening of Concrete Members Using Bonded FRP Systems

The high demand for durable, cost-effective and efficient strengthening methods has resulted in rapid growth in the use of externally bonded FRP sheets for strengthening of concrete structures. One of the most extensive uses of bonded FRP sheets has been for seismic upgrading of concrete structures. Wrapping of columns and bridge piers with FRP sheets for increased ductility and strengthening is a technique that has recently been widely researched and the number of field applications is growing rapidly. The use of FRP sheets and strips for flexural strengthening and stiffening of beams and slabs has also been studied extensively and is increasingly being applied to rehabilitate existing concrete structures. In comparison with the other strengthening applications described above, shear strengthening of concrete beams and girders using FRP sheets has received less attention and is just now expanding in terms of research and field applications. (Triantafillou 1998)

The bond performance of FRP strengthening systems has been investigated for column wrapping, flexural strengthening and shear strengthening applications. For column wrapping applications, the sheets encircle the entire cross section and the ends are overlapped and bonded together. The success of this strengthening technique does not depend on the quality of the existing concrete substrate for load transfer, but rather, on the ability of the FRP system to remain bonded to itself until its ultimate capacity is reached. In flexural strengthening applications, load transfer between the concrete and the FRP strengthening system is critical and premature failure may occur in the zone of
high shear stress near the supports. Several design methods as well as techniques for anchoring the ends of the FRP sheets have been developed to avoid this premature failure. Due to the very short bond length available on the web of the beam in comparison with the underside of the beam, bond between the FRP sheet and the concrete is even more critical for shear strengthening applications.

Because bond between the FRP sheet and the concrete plays a key role in the load-carrying capacity of concrete members strengthened for both flexure and shear, the amount of research being conducted on the bond properties of FRP sheets is increasing significantly, as discussed later in this chapter. Mechanical anchorage of FRP sheets and strips is another alternative to alleviate potential bond problems in both shear strengthening and flexural strengthening applications.

2.2 BOND PROPERTIES OF FRP SHEETS

Failure of the bond mechanism between the FRP sheet and the concrete may occur in several different bond failure modes as described by Karbhari (1995) in Figure 2-1. Debonding may occur at either the FRP-adhesive interface or the adhesive-concrete interface, or on an alternating crack path between these interfaces. Cohesive failure within the adhesive layer may also occur, however the most predominant mode of bond failure is shear-tension failure within the concrete substrate.
The first two types of failure, debonding and cohesive failure could be considered failures of the FRP system applied to the concrete, while failure within the concrete suggests that the best possible use of the system has been realized. The preparation of the concrete substrate is therefore important and the condition of the existing concrete is the limiting factor for the strengthening technique if the FRP system has been properly selected and applied. The shear stress applied to the concrete substrate can be reduced by mechanically anchoring the sheets, and relying on the anchor plate and bolt to carry some of the bond stress, as shown in Figure 2-2. (Sato et al. 1997a)

2.2.1 Effect of Surface Preparation and Priming on Bond Performance

In order to achieve a good bond, the surface of the concrete must be clean, dry and free of all loose material. Typical methods for concrete surface preparation prior to the application of externally bonded FRP systems are: grinding; mechanical abrasion with a wire wheel; sand-blasting; and hydro-blasting with high pressure water. In separate bond tests, Chajes et al. (1996) found that mechanical abrasion with a wire wheel resulted in better bond performance than the grinding technique, while Yoshizawa et al. (1996) found that high pressure hydro-blasting increased the bond strength by a factor of two when compared with grinding.

For applications using FRP sheets, after the surface is clean, dry and free of all loose material, a primer is applied to strengthen the concrete surface. The primer is usually chemically similar to the impregnation resin for good adhesion to the resin, but less
viscous for good penetration into the concrete. In bond tests conducted by Yoshizawa et al. (1996), the use of different primers had no noticeable influence on the bond performance. The layer of concrete substrate which has been penetrated by the primer resin forms a composite material termed the "inter-phase" by Sato et al. (1997c). When the "inter-phase" is in the plastic condition, a limited zone of constant shear flow occurs and is called the effective bond length. (Sato et al. 1997c)

2.2.2 Effect of the Adhesive on Bond Performance

Arduini et al. (1997) conducted tests using the specimens shown in Figure 2-3 and concluded that, for the particular FRP strip system used, the shear strength at the concrete-adhesive interface was about 5 MPa and that the shear strength of the FRP-adhesive interface was about 3 times greater. The type of adhesive used by Arduini et al. had a tensile modulus of $E = 11 \text{ GPa}$ and an ultimate tensile strain of $\varepsilon_u = 3 \text{ millistrain}$. By comparison, Chajes et al. (1996) conducted single-lap bond tests using a low modulus adhesive, with $E = 0.2 \text{ GPa}$ and $\varepsilon_u = 600 \text{ millistrain}$, and found that the load transfer was so inefficient that failure was in the adhesive.

Hamada et al. (1997) conducted bond tests to compare the performance of flexible and rigid adhesive resins using the three-point bending bond specimens shown in Figure 2-4. Hamada et al. used a flexible adhesive with $E = 1.0 \text{ GPa}$ and $\varepsilon_u > 30 \text{ millistrain}$, and a rigid adhesive with $E = 3.15 \text{ GPa}$ and $\varepsilon_u = 8 \text{ millistrain}$. The modulus for both of these
adhesives lies between those reported for the tests conducted by Chajes et al. and Arduini et al.

As shown in Figure 2-5, Hamada et al. found that for the test specimens fabricated with the rigid adhesive, the maximum load increased with increasing stiffness of the FRP. By comparison, no such relationship was observed for the specimens with the flexible adhesive, and the two levels of load carrying capacity shown in Figure 2-5, were due to differences in the mode of failure. While the majority of specimens tested by Hamada et al. failed due to delamination, the flexible adhesive and rigid FRP sheet combination had the highest load carrying capacity and failure occurred within the concrete. (Hamada et al. 1997) These test results illustrate the beneficial effect of using an adhesive with a rupture strain greater than that of the high strength fibres.

Load is transferred from the FRP sheet or strip to the concrete through shear flow, and the relative stiffness of the FRP sheet or strip and the adhesive, influences how the load is transferred (Chajes et al. 1996). In spite of the wide range of adhesive mechanical properties used, general trends in bond behaviour have been observed for externally bonded FRP systems, and are discussed in the following sections.

### 2.2.3 Effect of FRP Sheet Stiffness on Bond Performance

Using the test specimens shown in Figure 2-6 (a), Maeda et al. (1997) investigated the effect of FRP sheet stiffness on the bond characteristics between FRP sheets and
concrete. Various bond lengths ranging from 65 mm to 700 mm were provided. An increase in the average bond strength with increasing stiffness in the FRP sheets was observed, as shown in Figure 2-6 (b). The average bond strength in Figure 2-6 (b) was calculated as the ultimate load divided by the bond length provided and the width of the FRP sheet. As illustrated in Figure 2-6 (b), the increase in average bond strength was not directly proportional to the increase in stiffness of the FRP sheet. An increase in the FRP stiffness by a factor of three resulted in an increase in average bond strength that was less than double. Iketani et al. (1997) and Yoshizawa et al. (1996) also reported an increase in bond strength with higher modulus FRP sheets and increased layers of sheets, for specimens similar to those tested by Maeda et al. (1997)

2.2.4 Effect of Concrete Strength on Bond Performance

The importance of good concrete surface preparation on bond performance was discussed earlier, and surface shear failure in the concrete substrate was described as indicative of the optimal performance of an externally bonded FRP system. However for design purposes, it is important to understand and quantify the relationship between the strength of the existing concrete substrate and the bond strength of the FRP system.

As shown in Figure 2-7 (a), Horiguchi and Saeki (1997) tested three different types of bond specimens with three different concrete compressive strengths, 11 MPa, 31 MPa and 46 MPa, for each type of specimen. An increase in bond strength with increased concrete strength was observed for all three types of specimens. However, the effect of
concrete strength on the bond performance was less for the shear type test compared with
the bending test and the tensile test, as shown in Figure 2-7 (b). Horiguichi and Saeki
proposed a relationship between the compressive strength of concrete and the bond
strength obtained for each type of test as follows:

\[
\begin{align*}
  f_b \text{ (shear)} & = 0.09 (f'_c)^{2/3} \\
  f_b \text{ (bend)} & = 0.22 (f'_c)^{2/3} \\
  f_b \text{ (tens)} & = 0.36 (f'_c)^{2/3}
\end{align*}
\]

Based on a limited number of specimens, Chajes et al. (1996) proposed a relationship
between the concrete compressive strength and the shear strength of the concrete, which
controls failure within the concrete substrate:

\[ v_{cu} = 0.09 (f'_c)^{1/2} \]  

(2-4)

The expressions for determining the bond strength shown in Equations (2-1) and (2-4) are
quite similar in spite of differences in the FRP material properties and the type of bond
specimens tested.

2.2.5 The Bond Mechanism and Effective Bond Length

Various researchers have reported that, beyond a bond length of 75 mm to 150 mm, the
ultimate load carrying capacity of bond specimens does not increase with an increase in
the available bond length provided, as shown in Figures 2-8 and 2-9 (Maeda et al. 1997, Iketani and Jinno 1996, Sato et al. 1997c, Yoshizawa et al. 1996). In tests by Chajes et al. (1996), the bond strength was fully developed within a length of 75 mm to 100 mm, although the maximum stress developed in the FRP strips at failure was only about one third of the ultimate tensile capacity of the strips.

The decrease in average bond strength with increasing bond lengths provided, illustrated in Figure 2-6 (b), is typical and indicates that all of the available bond length is not being utilized (Chajes et al. 1996). The term "effective bond length" is used to differentiate between the bond length provided and that portion of the bond length provided which is effective in transferring stresses between the concrete and the FRP sheet. The term "development length" is not appropriate unless the full tensile capacity of the FRP sheet can be developed within the effective bond length.

The length of FRP sheet over which stress transfer between the concrete and the FRP sheet occurs, that is the effective bond length, can be determined based on the distribution of strain along the length of the FRP sheet. A change in strain between two points along the sheet indicates that force is being transferred between the concrete and the FRP, while a constant non-zero strain between two points suggests that shear transfer has ceased or not yet begun.

The strain distribution recorded for an FRP strip tested by Chajes et al. (1996) is provided in Figure 2-10, and the strain distribution for FRP sheets tested by Maeda et. al. (1997)
are shown in Figures 2-11 (a) and 2-11 (b). The only difference between the two specimens tested by Maeda, are the 150 mm bond length provided for the specimen shown in Figure 2-11 (a) and the 300 mm bond length provided for the specimen shown in Figure 2-11 (b). Both specimens behave in a similar manner up to an applied load of 18 kN. For the specimen with a 300 mm bond length, the applied load is increased from 18 kN to an ultimate load of 23 kN, and the effective bond length shifts outward toward the end of the sheet. Although there is some increase in the maximum load, the increase is not proportional to the increase in bond length provided, and the concept of an effective bond length that shifts away from the crack is clearly illustrated.

Figure 2-12 illustrates a simple conceptual model proposed by Maeda et. al. (1997) to describe the shift in the effective bond length which was observed experimentally and is shown in Figure 2-11 (b). Brosens and Van Gemert (1997) present a similar idealization of the shear transfer behaviour, as shown in Figure 2-13, and suggest that the maximum shear stress shifts and increases slightly until the ultimate capacity is reached. Other researchers have described the failure of bonded FRP sheets as progressive, with the effective bond length transferring from the crack toward the end of the sheet (Iketani and Jinno 1997, Chajes et al. 1996).

The model proposed by Maeda et al. (1997) does not account for the small increase in capacity obtained with an increase in bond length provided, as shown in Figures 2-11 (a) and (b) and as proposed by Brosens and Van Gemert (1997). However, as mentioned previously and as shown in Figures 2-8 and 2-9, many researchers report only a very
small or negligible increase in capacity with an increase in bond length provided. (Maeda et al. 1997, Iketani and Jinno 1996, Sato et al. 1997c, Yoshizawa et al. 1996)

2.3 MODELS FOR BOND STRENGTH PREDICTION

It is difficult to develop general models for predicting bond strength due to the different types of bond specimens tested, the wide variety of FRP strengthening systems with different material properties that are used for the bond tests, and variation in the strength of the concrete used for bond tests. Many bond tests are conducted to provide specific bond performance information for a specific strengthening application. As will be discussed further in Chapter 3, the capacity of strengthened concrete members is often predicted based on bond performance data from specific bond tests which are conducted as part of the same experimental program.

Since bond performance controls the effectiveness of many strengthening applications, the design of concrete members strengthened with FRP sheets requires a general model capable of predicting the bond performance of the FRP system. As the database of bond tests grows, the potential for developing a general model for predicting bond strength also increases. In the following section, two models for predicting bond strength are described. Although based on specific experimental work conducted by each group of researchers, and including empirical constants, the models could be used to predict the bond performance of similar types of FRP sheets and strips used for other applications.
2.3.1 Model Based on Constant Strain Gradient and Effective Bond Length

Using the idealized model shown in Figure 2-12, Maeda et al. (1997) proposed a method to predict the bond strength of FRP sheets as a function of the FRP sheet rigidity. The model is based on the effective bond length, $L_e$, which is a function of the FRP sheet rigidity. The constant strain gradient, $de/dx$, which occurs along the effective bond length, $L_e$, is assumed equal to $0.0001102$ mm$^{-1}$ and is considered independent of the FRP sheet rigidity. The maximum strain that can be developed in the FRP sheets is calculated based on the constant strain gradient and the effective bond length, $L_e$, as follows:

$$\varepsilon_{f_{\text{max}}} = 0.0001102 \ L_e$$  \hspace{1cm} (2-5)

The bond strength, or maximum force in the FRP sheets at ultimate, $P_{f_{\text{max}}}$, can then be determined based on the maximum strain developed in the FRP sheets as follows:

$$P_{f_{\text{max}}} = \varepsilon_{f_{\text{max}}} \ E_r t_r w_f$$  \hspace{1cm} (2-6)

The relationship between the effective bond length, $L_e$, and the rigidity of the FRP sheets, $E_r t_r$, based on experimental results, is as follows:

$$L_e = \exp[6.134 - 0.580 \ln(E_r t_r)]$$  \hspace{1cm} (2-7)

Where, \( E_r \) = modulus of elasticity of FRP sheets in GPa

\( t_r \) = thickness of FRP sheets in mm
Figure 2-14 shows the proposed relationship graphically as well as the experimental test results which form the basis of the proposed model. The test results indicated that the increase in the ultimate load, $P_f \text{max}$, was not proportional to the increase in rigidity of the FRP sheets, resulting in a decreased effective bond length, $L_e$, for FRP sheets with a greater rigidity, as shown in Figure 2-14. Since the strain rate is assumed constant, the maximum strain, $\varepsilon_f \text{max}$, calculated using Equations (2-5) and (2-7) is therefore lower for FRP sheets with greater rigidity.

Sato et al. (1997c) conducted bond tests using the type of specimen shown in Figure 2-15 (a) which is similar to those tested by Maeda et al. A typical example of CFRP sheet strain distribution for the specimens tested by Sato et al. is shown in Figure 2-15 (b) and shows a trend in bond behaviour similar to the specimens tested by Maeda et al. Sato et al. report an average constant strain gradient in the effective bond length zone of 0.000168 mm\(^{-1}\). Chajes et al. (1996) observed a strain gradient of 0.000045 mm\(^{-1}\) for single-lap bond specimens.

### 2.3.2 The Fracture Mechanics Approach to Predicting Bond Strength

A model to predict the bond strength of elastic plate members externally bonded to concrete structures that was originally developed by Holzenkampfer and based on a fracture mechanics approach, has been modified by Neubauer and Rostasy (1997) to predict the bond strength of FRP strips. Neubauer and Rostasy (1997) conducted 51 bond tests using the double lap specimen shown in Figure 2-16 (a) and found that the
maximum tensile force developed in the plate, $T_{cm, \text{max}}$, did not increase for bond lengths greater than the effective bond length, $l_{e, \text{max}}$. The test results were used in a linear regression analysis to determine two empirical constant $k_b$. The predicted maximum tensile force is a function of the tensile strength of the surface concrete, $f_{ctm}$, and the model includes a term accounting for the width of the FRP strip.

Figure 2-16 (b) shows results of the tests conducted by Neubauer and Rostasy compared to curves predicted using the model. The parameters FRP strip width, $b$, strip thickness, $t$, and concrete cube strength were varied, with specimens marked B25 having a concrete cube strength of 25 MPa and specimens marked B55 having a strength of 55 MPa. As shown in Figure 2-16 (b), the increased concrete strength resulted in a reduced effective bond length when comparing strips of equivalent thickness. Contrary to the model proposed by Maeda et al. however, this model predicts an increase in the effective bond length with an increase in FRP strip stiffness, as shown in Figure 2-16 (b).

The model proposed by Neubauer and Rostasy is based on a fracture mechanics approach for sliding or mode II concrete failure, the primary failure mode for all of the test specimens. However, Neubauer and Rostasy (1997) indicate that the vertical displacement of rough crack faces may create mode I or peeling stresses transverse to the concrete surface, which may explain the secondary effects of interlaminar FRP strip failure which were also observed. Investigation of the mode I peeling mechanism in combination with the mode II sliding mechanism has been initiated by Karbhari and Engineer (1996) with their development of the peel test shown in Figure 2-17.
1 - Peel Failure into Concrete
2 - Interfacial Failure Between Concrete and Adhesive
3 - Cohesive Failure in the Adhesive
4 - Interfacial Crack Between Adhesive and Composite
5 - Alternating Crack Between the two Interfaces

Figure 2-1  Bond Failure Modes (Karbhari 1995)

Figure 2-2  Bond Stress at Mechanical Anchorage for FRP Sheet (Sato et al. 1997)

Figure 2-3  Tension-Shear and Compression-Shear Concrete-Adhesive Specimens Tested by Arduini et al. (1997)
Figure 2-4  Bending Bond Specimens Tested by Hamada et al. (1997)

Figure 2-5  Maximum Load vs FRP Sheet Stiffness: Bond Tests by Hamada et al. (1997)
Figure 2-6 (a) Bond Specimens Tested by Maeda et al. (1997)

Figure 2-6 (b) Bond Strength vs Bond Length Provided: Tests by Maeda et al. (1997)
Figure 2-7 (a) Three Types of Bond Specimens Tested by Horiguichi and Saeki (1997)

Figure 2-7 (b) Bond Strength vs Concrete Strength for Three Types of Bond Specimens Tested by Horiguichi and Saeki (1997)
Figure 2-8  Maximum Load vs Bond Length Provided:
Bond Tests by Maeda et al. (1997)

Figure 2-9  Maximum Load vs Bond Length Provided:
Bond Tests by Iketani and Jinno (1996)
Figure 2-10  Strain Distribution for Bond Tests Conducted by Chajes et al. (1996)

Figure 2-11 (a) Strain Distribution in Specimen No. 2 Tested by Maeda et al. (1997)

Figure 2-11 (b) Strain Distribution in Specimen No. 3 Tested by Maeda et al. (1997)
Figure 2-12 Schematic Strain Distribution for Model by Maeda et al. (1997)

Figure 2-13 Schematic Shear Stress Distribution Before and After Cracking (Brosnens and Van Gemert 1997)

Figure 2-14 Effective Bond Length vs FRP Sheet Stiffness (Maeda et al. 1997)
Figure 2-15 (a) Bond Specimens Tested by Sato et al. (1997c)

Figure 2-15 (b) Strain Distribution for Bond Tests by Sato et al. (1997c)
Figure 2-16 (a) Double-Lap Bond Specimen Tested by Neubauer and Rostasy (1997)

Figure 2-16 (b) Normalized Bond Test Results vs Bond Length Provided (Neubauer and Rostasy 1997)
Figure 2-17 Schematic of Peel Test Conducted by Karbhari and Engineer (1996)
3.1 GENERAL

While the use of FRP sheets or strips for flexural strengthening of concrete structures has been studied extensively, the study of externally bonded FRP sheets for shear strengthening has been limited and somewhat controversial Triantafillou (1998). This is not surprising, since shear behaviour in reinforced concrete beams, even without FRP strengthening, is a complex mechanism and the many existing models have not yet been unified and simplified for use in the design codes. (ACI-ASCE 1998)

It is well known that shear failure is generally brittle and catastrophic, with very little warning of impending failure. For this reason, it is standard practice to design concrete beams to ensure that a more ductile flexural failure would occur prior to any possible shear failure. Many concrete beams, therefore, require strengthening to increase the ultimate shear capacity beyond the ultimate flexural capacity. Some beams that require
Flexural strengthening may also require shear strengthening in order to fully develop the increased flexural capacity.

Various configurations of FRP sheets for shear strengthening of concrete members have been investigated in the literature. While the majority of researchers have applied the sheets to both sides of the beam as well as the underside, some researchers have applied sheets to the sides only. For some applications such as columns and bridge piers, it is possible to wrap the entire cross section for enhanced shear capacity. However, for most beams and bridge girders, the presence of the top slab or bridge deck makes it difficult to encase the entire cross section. In order to improve the efficiency of FRP sheets used for shear strengthening applications, interest has been increasing in the use of mechanical anchorage at the top of the beam web, just below the slab. Figure 3-1 provides an illustration of the typical methods used to strengthen concrete sections in shear.

The contribution of the FRP sheets to the shear capacity of the beam at ultimate depends upon the controlling failure mechanism. Shear failure may occur due to failure of the bond mechanism between the FRP sheet and the concrete or by tensile fracture of the sheet. Fracture of the FRP sheet is more likely to occur when there is sufficient anchorage of the sheet, either by complete wrapping of the cross section, or by the use of mechanical anchorage. Because stress concentrations may occur even at well-rounded corners, the stress causing rupture of the FRP sheets is typically lower than the ultimate strength of the sheet. As discussed in Chapter 2, bond failure also results in maximum FRP strain levels that are much lower than the rupture strain of the material.
Many of the first models for predicting the shear capacity of beams strengthened with FRP sheets were based solely on the specific experimental studies from which they came, often considering only a single failure mode that was observed in a small number of tests with a limited variety of parameters. Most of the first analytical models showed good agreement because they were calibrated using the same set of data that was used for verification of the model (Triantafillou 1998). Recently however, the experimental database available in the literature has increased sufficiently for more generalized models to be developed. Three recently developed generalized models for predicting shear capacity, based on three different approaches, are discussed in this chapter.

There are a wide variety of models available for predicting the shear behaviour of reinforced and prestressed concrete beams, even without considering FRP strengthening techniques. Therefore, some background on the shear models typically used as a basis for predicting the shear capacity of beams strengthened with FRP sheets is also provided in this chapter.

3.2 SHEAR BEHAVIOUR OF PRESTRESSED CONCRETE BEAMS

Prior to cracking in a reinforced concrete beam, the shear reinforcement does not contribute significantly to the shear resistance of the member. At this stage, the tensile strain in the shear reinforcement is very low, equivalent to the tensile strain in the concrete. Cracking occurs when the principal tensile stress in the concrete exceeds the tensile strength of the concrete.
The principal stress trajectories for a typical reinforced concrete beam just prior to cracking are shown in Figure 3-2 (a). Because the beam also resists moment, the inclination of the principal strains varies over the depth of the section. In a zone of relatively high moment, inclined shear cracks may occur as an extension of vertical flexural cracks and are called flexure-shear cracks. Inclined shear cracks may also initiate in the thin web of a girder and are called web shear cracks. For most reinforced concrete beams, the initial inclined shear cracks occur at 45 degrees, while for prestressed concrete beams, the presence of compressive prestressing forces results in much lower crack inclination angles. (ACI-ASCE 1998, Ramirez and Breen 1991)

After the initiation of inclined shear cracks, the shear resisting mechanisms in a reinforced or prestressed concrete member are very complex and cannot be predicted based on elastic beam theory. With an increase in the applied shear load, new shear cracks form while the existing cracks propagate and change inclination.

The 45 degree truss analogy that was introduced by Ritter in 1899 and developed further by Morsch in 1902, provides a conceptual model for the flow of internal forces in a cracked reinforced or prestressed concrete member that has formed the basis for most major design codes. (ACI-ASCE 1998) In the truss analogy, the concrete between the inclined shear cracks is idealized as diagonal compressive struts which push apart the top and bottom chords of the truss, while the tensile forces in the shear reinforcement pull the chords together, as shown in Figure 3-2 (b).
For all shear models based on the truss analogy, the contribution of the shear reinforcement is calculated based on the equilibrium of vertical forces, as shown in Figure 3-3 (a) and as follows:

\[
V_s = A_{sv} f_y z (\cot \theta) \tag{3-1}
\]

The term "z (cot \(\theta\))" represents the horizontal projection of one truss panel, from the top of one diagonal to the top of the next diagonal. If the shear reinforcement is inclined at an angle, \(\alpha\), then the horizontal projection of one truss panel becomes "z (cot \(\theta + \cot \alpha\))," as shown in Figure 3-3 (b). In the case of inclined shear reinforcement, only the vertical component of the force in the reinforcement is considered for vertical equilibrium.

Extensive experimental work has shown that the 45 degree truss analogy is overly conservative because it does not account for the following shear resisting mechanisms (ACI-ASCE 1998):

1. Shear stresses in the uncracked concrete of the flexural compression zone
2. Dowel action of the longitudinal reinforcement
3. Arch action
4. Shear friction at the crack interface or aggregate interlock
5. Residual tensile stresses that may be transmitted directly across shear cracks
The final two shear resisting mechanisms listed above are interrelated and result in a vertical component of force occurring along the shear crack. The angle of the principal compressive stress in the web is therefore less than the angle of the shear crack, which suggests that some additional contribution from the concrete should be considered in the equilibrium considerations for the truss.

In order to account for some of the additional shear resisting mechanisms listed above, and to compensate for the overly conservative nature of the 45 degree truss analogy, two basic approaches have been taken. As discussed in Section 3.2.1, one approach is to supplement the shear resistance provided by the shear reinforcement with an additional concrete contribution term, $V_c$. The second approach, described in Section 3.2.2, is to use the truss analogy with a variable compressive strut inclination angle, $\theta$. In Section 3.2.3, the use of the variable angle truss approach in combination with a concrete contribution is discussed.

The use of a lower value for the strut inclination angle, $\theta$, results in an increase in the contribution provided by the shear reinforcement, as calculated using Equation (3-1). The use of a strut inclination angle that is lower than the cracking angle, $\theta_{cr}$, can be justified by considering some of the additional shear resisting mechanisms listed previously. By providing additional vertical force components, these additional mechanisms result in a lower angle for the principal compressive stresses in the web.

The compression field theory and modified compression field theory are rational models
capable of predicting the strut inclination angle, $\theta$, based on the deformation of the shear reinforcement, the longitudinal reinforcement, and the inclined concrete compressive struts. By considering equilibrium, compatibility and the stress-strain relationship of each material, the response of a reinforced concrete member subjected to shear can be predicted throughout the entire range of loading. Recognizing the beneficial effects of tensile stresses existing in the cracked concrete, Vecchio and Collins (1986) modified the compression field theory further, as discussed in Section 3.2.4.

Most shear design codes recognize that with excessively wide shear cracks, several of the additional shear resisting mechanisms listed previously are no longer effective. Limitations on shear crack widths are therefore imposed, either directly or indirectly, by limiting the strain in the shear reinforcement. With excessive amounts of shear reinforcement, the compressive strength of the concrete struts may be exceeded prior to yielding of the shear reinforcement. Most shear design codes limit the amount of shear reinforcement to avoid this extremely brittle failure mode.

3.2.1 **Truss Model with Concrete Contribution**

Several design codes such as the ACI Building Code Requirements for Reinforced Concrete, the CSA Simplified Method and Eurocode 2 Standard Method, all include shear design procedures based on the truss model with the crack inclination angle, $\theta_{cr}$, and the compressive strut inclination angle, $\theta$, both equal to 45 degrees. The concrete contribution, $V_c$, is based on the shear load at which inclined cracking begins, and is
added to the shear reinforcement contribution to predict the nominal shear resistance of the beam as follows:

\[ V_n = V_c + V_s \] (3-2)

For prestressed concrete members, \( V_c \) is assumed to be the smaller of the shear load causing web shear cracks, \( V_{cw} \), and the shear load causing flexure-shear cracking, \( V_{ci} \). According to the ACI design code, the following equations can be used to predict the onset of shear cracking (ACI-318, 1995):

\[ V_{cw} = 0.3 \left( \sqrt{\frac{f_{c}'}{f_{pc}}} b_w d + V_p \right) \] (3-3)

\[ V_{ci} = 0.05 \left( \sqrt{\frac{f_{c}'}{f_{pc}}} b_w d + V_d + V_i \frac{M_{cr}}{M_{max}} \right) \] (3-4)

Both equations are based on predicting the level of applied shear load causing principal tensile stresses in the concrete that exceed the tensile strength of the concrete.

### 3.2.2 Variable Angle Truss Model

In addition to the Standard Method, the Eurocode 2 design code (1991) also includes a variable strut inclination method for predicting the shear capacity of reinforced concrete members. For beams with low amounts of shear reinforcement, the strut inclination angle is very low. Therefore, lower limits are set for \( \theta \), to ensure that a minimum amount
of shear reinforcement is used. The strut inclination angle, $\theta$, is allowed to vary between 22 and 68 degrees for beams with constant longitudinal reinforcement or 27 and 63 degrees for beams with curtailed longitudinal reinforcement. There is no additional concrete contribution considered with this model, although the method does require a check to ensure that crushing of the compressive struts does not occur.

Based on the theory of plasticity, crushing of the concrete struts simultaneously with yielding of the shear reinforcement is the limiting condition for a given amount of shear reinforcement. For a given amount of shear reinforcement, there is a value for the strut inclination angle, $\theta$, associated with this failure envelope, as shown in Figure 3-4. The limits for $\theta$, as set by the code, are also indicated in Figure 3-4. For comparison, the shear capacity predicted using the 45 degree truss model with a concrete contribution is also shown in Figure 3-4 for varying amounts of shear reinforcement.

3.2.3 Variable Angle Truss Model with Concrete Contribution

The modified sectional-truss model introduced by Ramirez and Breen (1991) includes a concrete contribution in combination with the variable angle truss model. In this model, the shear resistance of a reinforced or prestressed concrete member, $V_n$, is predicted using Equation (3-2) where the resistance provided by the shear reinforcement, $V_s$, is determined using Equation (3-1). Similar to the variable angle truss model included in Eurocode 2, lower limits for the strut inclination angle, $\theta$, are set at 30 degrees for
reinforced concrete beams and 25 degrees for prestressed concrete members. (Ramirez and Breen 1991)

The concrete contribution, \( V_c \), is included in the model in order to avoid the overly conservative results that are obtained when applying the variable angle truss model to beams experiencing low levels of shear stress. (Ramirez and Breen 1991) The concrete contribution proposed for reinforced concrete beams is shown in Figure 3-5 (a), and decreases with increasing levels of shear stress. The model recognizes the contribution of some of the additional shear resisting mechanisms listed previously, and acknowledges that some of these shear resisting mechanisms may be ineffective at higher levels of shear stress. (Ramirez and Breen 1991)

Ramirez and Breen (1991) suggest that although some of the additional shear resisting mechanisms decrease with increased shear stress, others such as the shear stress in the flexural compression zone are quite significant at high levels of shear stress. The presence of prestressing improves the sustained effectiveness of the additional shear resisting mechanisms even further. Therefore, the concrete contribution recommended for prestressed concrete beams does not decrease with increasing levels of shear stress, as shown in Figure 3-5 (b).

The following expression is introduced to evaluate the concrete contribution for prestressed concrete beams: (Ramirez and Breen 1991)
\[ V_c = K \cdot (0.166) \cdot \sqrt{f_c'} \cdot b_w \cdot d \]  \hspace{1cm} (3-5)

Where,

\[ K = \sqrt{1 + \frac{f_{pc}}{f_t}} \] and \[ 1.0 \leq K \leq 2.0 \]

\[ f_t = 0.166 \cdot \sqrt{f_c'} \] \hspace{1cm} (f'\text{ in MPa})

\[ f_{pc} = \text{compressive stress at neutral axis} \]

Similar to the ACI expression for \( V_{cw} \), Equation (3-5) above predicts the initiation of web shear cracking. For those sections where flexural cracking followed by flexure-shear cracking will initiate prior to web shear cracking, a value of 1.0 is used for \( K \). Although similar to the ACI approach for determining \( V_c \), Equation (3-5) is more conservative, as shown in Figures 3-6 (a) and (b). Figures 3-6 (a) and (b) provide a comparison of the ACI method and the modified sectional-truss method applied to predict the concrete contribution for the test beams and loading arrangements used in this experimental program.

3.2.4 Compression Field Theories

The compression field theory and modified compression field theory are based on compatibility of the deformations in each member of the idealized truss, utilizing the stress-strain relationship of each material, and satisfying equilibrium. The modified compression field theory refines the model even further by considering the beneficial effect of tensile stresses in the cracked concrete. (Vecchio and Collins 1986)
For the compression field theory, the model is formulated by considering equilibrium of a beam cross section subjected to shear only, as shown in Figure 3-7 (a). The applied shear force, \( V \), is resisted by the compressive stress in the diagonal concrete struts, \( f_2 \), and the additional tensile force due to shear only, \( N_v \), which is shared equally between the top and bottom chords of the truss. By considering vertical equilibrium of the cross section shown in Figure 3-7 (a), the following expression is derived for the applied shear force:

\[
V = f_2 b_w d_v \cos \theta \sin \theta \quad (3-6 \text{ a})
\]

Based on equilibrium in the horizontal direction, the following expression for the additional tensile force, \( N_v \), is derived:

\[
N_v = V \cot \theta \quad (3-6 \text{ b})
\]

The force in an individual stirrup is determined using the following expression, which is based on vertical equilibrium of the portion of the beam shown in Figure 3-7 (b):

\[
f_{sv} A_{sv} = f_2 b_w s \sin^2 \theta \quad (3-6 \text{ c})
\]

In addition to the three equilibrium equations given above, two compatibility equations are used to determine the average strain in the truss members and the strut inclination
angle, $\theta$. The term "average" strain, indicates that these strains apply to base lengths which are long enough to include several shear cracks. The compatibility equations are summarized by the Mohr's Circle of strains shown in Figure 3-8 and are as follows:

$$\tan^2 \theta = \frac{\varepsilon_x - \varepsilon_2}{\varepsilon_y - \varepsilon_2} \quad (3-7 \text{ a})$$

$$\varepsilon_1 = \varepsilon_y + \varepsilon_x - \varepsilon_2 \quad (3-7 \text{ b})$$

Where,

- $\varepsilon_1$ = principal tensile strain in web
- $\varepsilon_2$ = principal compressive strain in web
- $\varepsilon_x$ = strain in longitudinal direction in web
- $\varepsilon_y$ = strain in transverse (vertical) direction in web

The compatibility and equilibrium requirements are interrelated by considering the stress-strain relationships of the steel reinforcement in tension and the concrete in compression. A large number of experimental studies have indicated that the compressive strength of diagonally cracked concrete decreases with an increase in the principal tensile strain, $\varepsilon_1$. The following expression was introduced by Vecchio and Collins, to determine the maximum compressive strength of the concrete struts: (ACI-ASCE 1998)

$$f_{2\text{ max}} = \frac{f_c'}{0.8 + 170 \varepsilon_1} \leq f_c' \quad (3-8 \text{ a})$$
The compressive stress in the concrete struts, $f_2$, can be determined based on the principal compressive strain, $\varepsilon_2$, as follows: (Vecchio and Collins 1986)

$$f_2 = f_{2 \text{ max}} \left[ 2 \left( \frac{\varepsilon_2}{\varepsilon'_c} \right) - \left( \frac{\varepsilon_2}{\varepsilon'_c} \right)^2 \right] \quad (3-8 \text{ b})$$

Using Equations (3-8 a) and (3-8 b), the stress-strain relationship of diagonally cracked concrete can be determined at various levels of principal tensile strain, $\varepsilon_1$, as illustrated in Figure 3-9.

The proportion of the strain in the longitudinal direction, $\varepsilon_x$, which is attributed to the longitudinal tensile force due to shear only, $N_v$, can be determined as shown in Figure 3-10 (a). If the beam is also subjected to moment, the proportion of the longitudinal strain, $\varepsilon_x$, due to moment will vary over the depth of the cross section as shown in Figure 3-10 (b). The longitudinal strain, $\varepsilon_x$, used in the compatibility Equations (3-7 a) and (3-7 b) is therefore based on the strain at a selected depth of the cross section and includes the effects of both moment and shear, as illustrated in Figures 3-10 (a) and (b). Using the longitudinal strain, $\varepsilon_x$, at the centre of gravity of the longitudinal flexural reinforcement provides the greatest strain and therefore the most conservative results. For prestressed concrete beams, the beneficial effects of prestressing, in reducing the longitudinal strain, $\varepsilon_x$, must be taken into account.

The typical mode of failure is yielding of the stirrups with simultaneous yielding of the
longitudinal reinforcement. For lower strut inclination angles, $\theta$, the stress in the concrete compressive struts and the tension in the longitudinal reinforcement are increased, while the stirrup requirements are reduced. The compressive stress in the concrete struts, $f_2$, must be checked to ensure that the maximum stress, $f_{2\ max}$, is not exceeded and that crushing of the struts does not occur prior to yielding of the stirrups.

In the modified compression field theory, the effect of tensile forces in the concrete, $f_1$, are accounted for as shown in Figures 3-11 (a) and (b). Similar to the approach described for the compression field theory, the three equilibrium equations can be derived using Figures 3-11 (a) and (b). By considering vertical equilibrium of the cross section shown in Figure 3-11 (a), the following expression is derived for the applied shear force:

$$V = (f_1 + f_2) b_w d_v \cos \theta \sin \theta$$

(3-9 a)

Based on equilibrium in the horizontal direction, the following expression for the additional tensile force due to shear only, $N_v$, is derived:

$$N_v = (f_2 \cos^2 \theta - f_1 \sin^2 \theta) b_w d_v$$

(3-9 b)

The force in an individual stirrup is determined using the following expression, which is based on vertical equilibrium of the portion of the beam shown in Figure 3-11 (b):
The compatibility equations, Equations (3-7 a) and (3-7 b), are also used in the modified compression field theory, as is the stress-strain relationship for diagonally cracked concrete in compression defined by Equations (3-8 a) and (3-8 b).

The stress-strain relationship for concrete in tension suggested by Collins and Mitchell (1997) is shown in Figure 3-12. The relationship after cracking is expressed as follows:

\[ f_i = \frac{0.33 \sqrt{f_c}}{1 + \sqrt{500} \varepsilon_1} \]  \hspace{1cm} (3-10)

Although only average stresses and strains are considered in the equilibrium, compatibility and stress-strain equations, it is recognized that failure may occur due to local rather than the average stresses. The difference in the local stresses occurring between the shear cracks is compared with those occurring at the shear crack, as shown in Figures 3-13 (a) and (b), respectively. Since equilibrium requires that the sum of vertical forces be equivalent in both Figure 3-13 (a) and Figure 3-13 (b), the following expression can be derived:

\[ f_i = A_{sv} (f_y - f_{sv}) + v_{ci} \tan \theta \]  \hspace{1cm} (3-11)
As shown in Figures 3-13 (a) and (b), at lower levels of applied shear load, the stress in the steel stirrups reaches yield at the crack location, but is reduced at locations away from the crack. It is assumed that near ultimate however, the tensile stress in the stirrups will have reached yield both at the crack and between the cracks. Therefore, near ultimate Equation (3-11) can be simplified as follows: (ACI-ASCE 1998)

\[ f_i = v_{ci} \tan \theta \] (3-12a)

The maximum shear stress that can occur at the shear crack, \( v_{ci} \), is related to the size of the maximum aggregate, \( a \), and the crack width, \( w \). The crack width is determined by multiplying the spacing between cracks, \( s_{cr} \), by the principal tensile strain, \( \varepsilon_1 \). Equation (3-12 a) can therefore be modified to form part of the stress-strain relationship shown in Figure 3-12 as follows: (Vecchio and Collins 1986)

\[ f_i \leq \frac{0.18 \sqrt{f_c'}}{0.3 + 24 s_{cr} \varepsilon_1 / (a + 16)} \tan \theta \] (3-12 b)

The modified compression field theory can be formulated to provide an expression similar to those used in the other shear prediction models. Equation (3-9 c) is written in terms of the compressive stress, \( f_2 \), and substituted into (3-9 a), resulting in the following expression:
\[ V = f_1 \cot \theta b_w d_v + A_{sv} f_y d_v \cot \theta \]

The first term in Equation (3-13) could be considered similar to the concrete contribution provided in other models, \( V_c \), while the second term is identical to the stirrup contribution \( V_s \), in Equation (3-1). The General Method for shear design provided in the CSA code includes an expression similar to Equation (3-13), where:

\[ f_1 \cot \theta = \beta \sqrt{f_c} \]

and

\[ V = \beta \sqrt{f_c} b_w d_v + A_{sv} f_y d_v \cot \theta \]

For shear design, values for \( \theta \) and \( \beta \) are selected from curves provided in the code, based on the level of applied shear stress to be resisted and the longitudinal strain, \( \varepsilon_x \). The code recommends, conservatively, that the value for the longitudinal strain, \( \varepsilon_x \), be taken at the centre of gravity of the longitudinal reinforcement.

3.3 SHEAR BEHAVIOUR OF BEAMS STRENGTHENED WITH FRP SHEETS

Similar to beams with only steel stirrups for shear reinforcement, the shear strength, \( V_n \), of a reinforced concrete beam with externally bonded FRP sheets can be calculated as the sum of the shear resisting contributions of the concrete, \( V_c \), the steel stirrups, \( V_s \), and the
FRP sheets, $V_{f_{\text{max}}}$. Based on the truss analogy, the contribution of the CFRP sheets can be determined by considering the equilibrium of vertical forces, as shown in Figure 3-14 and as follows:

$$V_f = \varepsilon_{f_{\text{ave}}} E_f 2n_r t_r w_r d_r (\cot \theta + \cot \alpha_f) \sin \alpha_f$$  \hspace{1cm} (3-15)

Where,

- $V_f$ = shear resistance provided by FRP sheets
- $\varepsilon_{f_{\text{ave}}}$ = average measured strain in FRP sheets
- $E_f$ = modulus of elasticity of FRP sheets
- $n_r$ = number of layers of FRP sheets per one side of beam
- $t_r$ = thickness of one layer of FRP sheets
- $w_r$ = width of one FRP sheet perpendicular to principle fibres
- $s_r$ = spacing of FRP sheets along longitudinal beam axis
- $d_f$ = depth of cross section over which FRP sheets are effective
- $\theta$ = angle of inclined shear crack
- $\alpha_f$ = angle of principle fibres of FRP sheets

The majority of models introduced to predict the shear capacity of beams strengthened in shear using CFRP sheets consist of equations similar to Equation (3-15). As will be described in the following sections, the main difference between each model is in the selection of the average or effective FRP strain, $\varepsilon_{f_{\text{ave}}}$, and the depth of the cross section.
over which the FRP sheets are effective, \( d_r \). Values for \( \varepsilon_{f,ave} \) and \( d_r \) depend on the mode of failure, and are determined based on the FRP sheet configuration and stiffness.

If sufficient anchorage of the FRP sheets is provided, either by complete wrapping of the cross section or by the use of mechanical anchorage, the typical mode of failure is rupture of the FRP sheet, as shown in Figure 3-15 (a). Because stress concentrations may occur even at rounded corners and at debonded areas around cracks, the stress causing rupture of the FRP sheets is typically lower than the ultimate strength of the sheet (Araki et al. 1997, Triantafillou 1998).

For the majority of shear strengthening applications, the presence of a slab supported by the beam prevents complete wrapping of the cross section. Without complete wrapping of the cross section or mechanical anchorage of the sheets, the predominant mode of failure is failure of the bond mechanism by shear-tension failure in the concrete substrate, as shown in Figure 3-15 (b) (Alexander and Cheng 1996, Al-Sulaimani et al. 1994, Drimoussis and Cheng 1994, Norris et al. 1997, Sato et al. 1996, Triantafillou 1998, Uji 1992).

3.3.1 Model for Shear Capacity Based on FRP Stiffness

One of the first generalized models that is based on a collection of experimental work reported in the literature and extending beyond one single experimental program, was developed by Triantafillou in 1998. The experimental test results for beams strengthened
with FRP sheets that were used to develop the model are summarized in Table 3-1. As shown in Table 3-1, both failure modes were observed, rupture of the CFRP sheets for the wrapped beams and failure of the bond mechanism for the beams with U-shaped sheets or sheets on the sides only. The only exception is the beams tested by Chajes et al. (1995), where rupture occurred for sheets that were not wrapped completely around the beam cross section.

Based on the ultimate shear failure load reported for each test beam, and using an expression similar to Equation (3-15), Triantafillou determined the effective strain in the FRP sheets at failure for each beam. Figure 3-16 shows the calculated effective strain in the FRP, $\varepsilon_{frp}$, as a function of the area fraction and stiffness of the FRP sheets $\rho_{frp} E_{frp}$.

The second order equation for the best-fit curve shown in Figure 3-16 is as follows:

If $0 \leq \rho_{frp} E_{frp} \leq 1$,

$$\varepsilon_{frp} = 0.0119 - 0.0205 (\rho_{frp} E_{frp}) + 0.0104 (\rho_{frp} E_{frp})^2$$  \hspace{1cm} (3-16 a)

And if $\rho_{frp} E_{frp} > 1$,

$$\varepsilon_{frp} = -0.00065 (\rho_{frp} E_{frp}) + 0.00245$$  \hspace{1cm} (3-16 b)

Where,

$$\rho_{frp} = 2t_r \frac{w_f}{b_w s_f}$$
As can be observed in Figure 3-16, the effective strain in the FRP sheets at failure is consistently greater for the beams with wrapped sheets when compared to those beams where shear failure occurred due to failure of the bond mechanism.

The variation in the FRP sheet contribution, $V_{frp}$, with increasing stiffness, $\rho_{frp}E_{frp}d$, was calculated using Equations (3-15) and (3-16) and plotted, along with the beam test results, in Figure 3-17 (Triantafillou 1998). The curve for $V_{frp}$ shown in Figure 3-17 suggests that there is a limiting area fraction of FRP, $\rho_{frp}$, beyond which there is no increase in the effectiveness of the FRP sheets for shear strengthening of the beam. This observation is similar to the trend observed by Maeda et al. (1997) for the simple bond tests, where a second layer of FRP sheets did not double the capacity of the bond specimen.

3.3.2 Predictions Based on Average Bond Stress

Because failure of the bond mechanism is a predominant mode of failure for beams shear strengthened with FRP sheets, many researchers have based predictions for the shear resistance provided by the FRP sheets on the average bond stress occurring over a given surface area of the beam. The bond area and the method for determining the average or maximum bond stress vary slightly for different researchers as discussed in this section.

Based on test results for beams strengthened with 3 mm thick glass fibre plates, Al-Sulaimani et al. (1994) suggest that the bond stress distribution can be described as
shown in Figure 3-18, and that the FRP shear resistance can be determined based on the average bond stress, $\tau_{ave}$, and the height of the FRP plates, $h$, as follows:

$$V_T = 2 F_r = 2 \tau_{ave} (d h/2)$$  \(3-17\)

Where, $F_r$ = force in FRP plate on one side of the beam

Equation (3-17) is used to predict the shear resistance provided by continuous FRP plates, while the shear resistance provided by strips of FRP plates can be calculated using the same equation multiplied by the ratio of the width of each strip to the strip spacing, $b_r/s_r$.

Al-Sulaimani et al. also conducted double-lap bond tests, and determined that the ultimate bond strength, $\tau_{ult}$, was 3.5 MPa for the specific materials used in their experimental program. Using the beam test results, the average bond stress, $\tau_{ave}$, was found to range from 0.8 to 1.2 MPa for the beams with FRP plates applied to the sides only. For the beams with U-shaped FRP jackets, Al-Sulaimani et al. recommend that the ultimate bond strength, $\tau_{ult}$, be substituted for the average bond stress when using Equation (3-17) to predict the FRP shear resistance.

Using a similar bond stress distribution and method for predicting the FRP shear contribution based on bond strength, Chaallal et al. (1998) provide a more generalized model by introducing an expression for calculating the ultimate bond strength. The ultimate bond strength is calculated based on Roberts’ approximate analytical solution,
and using the material properties of the FRP sheet and the adhesive. Chaallal et al. recommend the following relationship between the average bond stress and the calculated ultimate bond stress:

\[
\tau_{\text{ave}} = \tau_{\text{ult}} / 2
\]  

(3-18)

Sato et al. (1996) also observed bond failure above and below the shear crack for beams with 0.11 mm thick CFRP sheets applied both in U-shaped strips and strips on the sides of the beam only, as shown in Figure 3-19. The independent bond test results shown in Figure 3-20, were used to predict the shear resistance of the FRP sheets, based on the available bond length provided above or below the shear crack for each FRP strip. This approach recognizes the trend in bond test results, as discussed in Chapter 2, where average bond strength decreases with increasing bond lengths provided. Sato et al. report that the FRP shear resistance predicted using this method did not compare well with the beam test results, and suggest that the bond relationship requires further clarification.

### 3.3.3 Model for Shear Capacity Based on Effective Bond Length

As discussed in Chapter 2, various researchers have reported that beyond a certain "effective bond length," typically in the range of 75 to 150 mm, an increase in the available bond length provided does not significantly increase the ultimate capacity of bond specimens.
The concept of an effective bond length, $L_e$, is applied to beams strengthened in shear using FRP sheets as shown in Figures 3-21 (a), (b) and (c). For U-shaped FRP sheets, the portion of the sheet with an available bond length above the shear crack less than $L_e$, is considered ineffective. (Alexander and Cheng 1996, Khalifa et al. 1998) For sheets applied to the sides of the beam only, as shown in Figure 3-21 (b), the available bond length both above and below the shear crack is compared to $L_e$ in order to determine the portion of the FRP sheet that is effective in resisting shear forces. The concept of an effective bond length can also be applied to beams strengthened with diagonal FRP sheets, as shown in Figure 3-21 (c).

Based on the beams shown in Figures 3-21 (a) to (b), it is apparent that the depth of the cross section plays a significant role in the effectiveness of FRP sheets when bond failure is the controlling failure mechanism. If the depth of the cross section is too small, the available bond length above and below the shear crack may be less than the effective bond length, $L_e$, and the maximum FRP strain that could be developed in the FRP sheets may not be reached. The depth of the cross section should be carefully considered when examining test results reported for very small test beams.

Khalifa et al. (1998) have introduced a model for predicting the shear resistance of FRP sheets which considers both the effective bond length model proposed by Maeda et al. (1997), as well as the effective strain relationship shown in Equation (3-16) and proposed by Triantafillou (1998). The FRP shear resistance is calculated using Equation (3-15), and the lowest value obtained using the two different approaches is selected.
The bond strength model proposed by Maeda et al. (1997), as described in Section 2.3.1, is used to determine the effective bond length, \( L_e \), and the maximum strain that can be developed in the FRP sheets, \( \varepsilon_f_{\text{max}} \), as a function of their stiffness. As shown in Figures 3-21 (a), (b) and (c), the effective bond length is used to determine the portion of the FRP sheet which is effective in resisting shear forces. The effective depth of the FRP sheet, \( d_e \), is reduced based on the effective bond length and the configuration of the FRP sheets. The maximum strain that can be developed in the sheets, \( \varepsilon_f_{\text{max}} \), is assumed to be evenly distributed over the effective depth, \( d_e \), so that \( \varepsilon_f_{\text{max}} \) is used for the average FRP strain, \( \varepsilon_f_{\text{ave}} \), in Equation (3-15).

For design purposes, in order to maintain the integrity of the concrete contribution, Khalifa et al. also impose a restriction on the strain in the FRP sheets, limiting \( \varepsilon_f_{\text{ave}} \) to about 0.004 to 0.005.

### 3.3.4 Load Sharing Between FRP Sheets and Steel Stirrups

An accurate assessment of the load sharing relationship between the CFRP sheets and the steel stirrups is important for the design of strengthening schemes where each component is required to carry a portion of the applied loading. In separate investigations, Uji (1992) and Sato et. al. (1996) observed that the ratio of the shear resistance provided by the FRP sheets and the steel stirrups, \( V_{frp}/V_s \), was not based on equivalent strain in the
two materials, and therefore was not determined by the ratio of their stiffnesses $E_{frp}A_{frp}/E_sA_s$. Both Uji and Sato et al. suggest that this behaviour is due to the difference in the bond properties of the FRP sheets when compared with the stirrups.

Uji (1992) suggests that due to the superior bond performance of the FRP sheets, the FRP sheet elongation is localized around the shear crack, while the stirrup strains evenly over most of its length. Higher strains are therefore developed in the FRP sheets when compared to the stirrups at the same crack location (Uji 1992). Miyauchi et al. (1997) also reported that the strain in the FRP sheets was higher than the strain in the stirrups for reinforced concrete beams strengthened with CFRP sheets.

As shown in Figure 3-22, Sato et al. (1996) plotted the variation in the ratio of the contributions, $V_{frp}/V_s$, for increasing levels of applied shear load, for two test beams with different steel shear reinforcement ratios, $\rho_w$. The dotted and solid lines in Figure 3-22 show the stiffness ratio, $E_{frp}A_{frp}/E_sA_s$, for each beam. As illustrated by Figure 3-15, the ratio of the contributions, $V_{frp}/V_s$, decreases with increasing load and approaches a constant level that is approximately 1.3 times greater than the ratio of the stiffnesses. Uji (1992) reported a similar trend, as the debonded area of the FRP sheet increased with increasing levels of applied load, the ratio of the contributions, $V_{frp}/V_s$, approached the ratio of the stiffnesses, $E_{frp}A_{frp}/E_sA_s$. 
Table 3-1 Summary of Experimental Results: Beams Strengthened with FRP Sheets

<table>
<thead>
<tr>
<th>Beam*</th>
<th>$b_{w,m}$</th>
<th>$d_m$</th>
<th>FRP type†</th>
<th>$\rho_{pp}$</th>
<th>$E_{pp},$ GPa</th>
<th>$\beta,$ deg</th>
<th>$\varepsilon_{pp,e}$</th>
<th>Failure mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>B(3)</td>
<td>0.114</td>
<td>0.085</td>
<td>G, sides (s)</td>
<td>0.011</td>
<td>16.8</td>
<td>45</td>
<td>0.0066</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>B(4)</td>
<td>0.114</td>
<td>0.085</td>
<td>G, s</td>
<td>0.027</td>
<td>16.8</td>
<td>45</td>
<td>0.0056</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>U(3)</td>
<td>0.1</td>
<td>0.17</td>
<td>C, wrap</td>
<td>0.00194</td>
<td>230</td>
<td>90</td>
<td>0.0050</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>U(5)</td>
<td>0.1</td>
<td>0.17</td>
<td>C, s</td>
<td>0.00194</td>
<td>230</td>
<td>90</td>
<td>0.0030</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>U(6)</td>
<td>0.1</td>
<td>0.17</td>
<td>C, s</td>
<td>0.00194</td>
<td>230</td>
<td>90</td>
<td>0.0034</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>U(7)</td>
<td>0.1</td>
<td>0.17</td>
<td>C, s</td>
<td>0.00399</td>
<td>230</td>
<td>90</td>
<td>0.0015</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>D(F2)</td>
<td>0.038</td>
<td>0.127</td>
<td>A, sides &amp; bottom (s &amp; b)</td>
<td>$\rho_{pp}E_{pp} = 0.363$</td>
<td>90</td>
<td>&gt;0.0044</td>
<td>Flexure</td>
<td></td>
</tr>
<tr>
<td>A(WO)</td>
<td>0.15</td>
<td>0.113</td>
<td>G, s</td>
<td>0.04</td>
<td>16</td>
<td>90</td>
<td>0.0008</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>A(SO)</td>
<td>0.15</td>
<td>0.113</td>
<td>G, s</td>
<td>0.016</td>
<td>16</td>
<td>90</td>
<td>0.0018</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>A(IO)</td>
<td>0.15</td>
<td>0.113</td>
<td>G, s &amp; b</td>
<td>0.04</td>
<td>16</td>
<td>90</td>
<td>&gt;0.0016</td>
<td>Flexure</td>
</tr>
<tr>
<td>O(BS12)</td>
<td>0.18</td>
<td>0.36</td>
<td>C, wrap</td>
<td>0.0012</td>
<td>230</td>
<td>90</td>
<td>0.0084</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>O(BS24)</td>
<td>0.18</td>
<td>0.36</td>
<td>C, wrap</td>
<td>0.00245</td>
<td>230</td>
<td>90</td>
<td>0.0062</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>O(BM06)</td>
<td>0.18</td>
<td>0.36</td>
<td>C, wrap</td>
<td>0.0006</td>
<td>230</td>
<td>90</td>
<td>0.0117</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>O(BM12)</td>
<td>0.18</td>
<td>0.36</td>
<td>C, wrap</td>
<td>0.0013</td>
<td>230</td>
<td>90</td>
<td>0.0099</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>O(BM18)</td>
<td>0.18</td>
<td>0.36</td>
<td>C, wrap</td>
<td>0.0019</td>
<td>230</td>
<td>90</td>
<td>0.0078</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>O(BM24)</td>
<td>0.18</td>
<td>0.36</td>
<td>C, wrap</td>
<td>0.0025</td>
<td>230</td>
<td>90</td>
<td>0.0069</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>O(BL06)</td>
<td>0.18</td>
<td>0.36</td>
<td>C, wrap</td>
<td>0.0006</td>
<td>230</td>
<td>90</td>
<td>0.0056</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>O(BL12)</td>
<td>0.18</td>
<td>0.36</td>
<td>C, wrap</td>
<td>0.0019</td>
<td>230</td>
<td>90</td>
<td>0.0084</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>O(BMW06)</td>
<td>0.18</td>
<td>0.36</td>
<td>C, wrap</td>
<td>0.0013</td>
<td>230</td>
<td>90</td>
<td>0.0069</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>O(BMW12)</td>
<td>0.18</td>
<td>0.36</td>
<td>C, wrap</td>
<td>0.0024</td>
<td>230</td>
<td>90</td>
<td>0.0046</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>O(BMW24)</td>
<td>0.18</td>
<td>0.36</td>
<td>C, wrap</td>
<td>0.0025</td>
<td>230</td>
<td>90</td>
<td>0.0120</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>O(2)</td>
<td>0.4</td>
<td>0.34</td>
<td>C, wrap</td>
<td>0.00029</td>
<td>230</td>
<td>90</td>
<td>0.0103</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>O(3)</td>
<td>0.4</td>
<td>0.34</td>
<td>C, wrap</td>
<td>0.00038</td>
<td>230</td>
<td>90</td>
<td>0.0049</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>C(A)</td>
<td>0.0635</td>
<td>0.1525</td>
<td>A, s &amp; b</td>
<td>0.033</td>
<td>11</td>
<td>90</td>
<td>0.0049</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>C(E)</td>
<td>0.0635</td>
<td>0.1525</td>
<td>G, s &amp; b</td>
<td>0.021</td>
<td>14.3</td>
<td>90</td>
<td>0.0063</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>C(G)</td>
<td>0.0635</td>
<td>0.1525</td>
<td>C, s &amp; b</td>
<td>0.018</td>
<td>21</td>
<td>90</td>
<td>0.0052</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>C(45G)</td>
<td>0.0635</td>
<td>0.1525</td>
<td>C, s &amp; b</td>
<td>0.018</td>
<td>21</td>
<td>45</td>
<td>0.0051</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>M(B2)</td>
<td>0.127</td>
<td>0.133</td>
<td>C, s &amp; b</td>
<td>$\rho_{pp}E_{pp} = 0.409$</td>
<td>90</td>
<td>&gt;0.0020</td>
<td>Flexure</td>
<td></td>
</tr>
<tr>
<td>S(S2)</td>
<td>0.2</td>
<td>0.26</td>
<td>C, s</td>
<td>0.006</td>
<td>230</td>
<td>90</td>
<td>0.0010</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>S(S3)</td>
<td>0.2</td>
<td>0.26</td>
<td>C, s &amp; b</td>
<td>0.006</td>
<td>230</td>
<td>90</td>
<td>0.0017</td>
<td>Shear (debonding)</td>
</tr>
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<td>S(S4)</td>
<td>0.2</td>
<td>0.26</td>
<td>C, s</td>
<td>0.012</td>
<td>230</td>
<td>90</td>
<td>0.0005</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>S(S5)</td>
<td>0.2</td>
<td>0.26</td>
<td>C, s &amp; b</td>
<td>0.012</td>
<td>230</td>
<td>90</td>
<td>0.0008</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>S(S6)</td>
<td>0.2</td>
<td>0.26</td>
<td>C, s</td>
<td>0.012</td>
<td>230</td>
<td>90</td>
<td>&gt;0.0009</td>
<td>Flexure</td>
</tr>
<tr>
<td>T(S1a)</td>
<td>0.07</td>
<td>0.10</td>
<td>C, s</td>
<td>0.0022</td>
<td>235</td>
<td>90</td>
<td>0.0041</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>T(S1b)</td>
<td>0.07</td>
<td>0.10</td>
<td>C, s</td>
<td>0.0022</td>
<td>235</td>
<td>90</td>
<td>0.0034</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>T(S2a)</td>
<td>0.07</td>
<td>0.10</td>
<td>C, s</td>
<td>0.0033</td>
<td>235</td>
<td>90</td>
<td>0.0032</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>T(S2b)</td>
<td>0.07</td>
<td>0.10</td>
<td>C, s</td>
<td>0.0033</td>
<td>235</td>
<td>90</td>
<td>0.0026</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>T(S3a)</td>
<td>0.07</td>
<td>0.10</td>
<td>C, s</td>
<td>0.0044</td>
<td>235</td>
<td>90</td>
<td>0.0020</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>T(S3b)</td>
<td>0.07</td>
<td>0.10</td>
<td>C, s</td>
<td>0.0044</td>
<td>235</td>
<td>90</td>
<td>0.0016</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>T(S1-45)</td>
<td>0.07</td>
<td>0.10</td>
<td>C, s</td>
<td>0.0022</td>
<td>235</td>
<td>45</td>
<td>0.0030</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>T(S2-45)</td>
<td>0.07</td>
<td>0.10</td>
<td>C, s</td>
<td>0.0033</td>
<td>235</td>
<td>45</td>
<td>0.0022</td>
<td>Shear (debonding)</td>
</tr>
<tr>
<td>T(S3-45)</td>
<td>0.07</td>
<td>0.10</td>
<td>C, s</td>
<td>0.0044</td>
<td>235</td>
<td>45</td>
<td>0.0013</td>
<td>Shear (debonding)</td>
</tr>
</tbody>
</table>

* B = Beroz; U = Uji; D = Dolan et al.; A = Al-Sulaimani et al.; O = Ohuchi et al.; C = Chajes et al.; M = Malvar et al.; S = Sato et al.
† T = present study (Triantafyllou)
|           |           |           |           |           |           | Shear (debonding) |

Symbols for each beam appear in parentheses, as assigned by those who conducted tests.

* FRP; C = CFRP; A = AFRP; sides = bonded to sides only; wrap = wrapped around.

1 m = 39.4 in.; 1 GPa = 145 ksi.
Figure 3-1 Typical Shear Strengthening Schemes Using Externally Bonded FRP Sheets
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(Triantafillou 1998)
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c.g. longitudinal reinforcement

Figure 3-21 (b) Effective Depth of FRP Sheets for Shear Strengthening on Sides Only

Figure 3-21 (c) Effective Depth of Diagonal U-Shaped FRP Sheets
Figure 3-22  Ratio of Contributions to Shear Resistance $V_{fr}/V_s$ vs Applied Load (Sato et al. 1996)
4.1 GENERAL

The objective of the experimental program was to investigate the use of externally bonded FRP sheets for shear strengthening of I-shaped prestressed concrete bridge girders. Seven scale model prestressed concrete girders were strengthened, using three different types of CFRP sheets for ten different CFRP configurations. The ten meter long beams were 1:3.5 scale models of the girders used for the Maryland Bridge in Winnipeg, Manitoba, Canada. The beams were tested to failure at each end to examine their behaviour and determine the most efficient strengthening scheme. Because the bond between the CFRP sheets and the concrete is a critical component of this strengthening method, a series of bond specimens were tested in order to determine the bond characteristics.

The first series of four beams was fabricated using internal shear reinforcement with a shape identical to that used in the Maryland Bridge and shown in Figure 4-1(a). In order to extend the applicability of the experimental results, the second series of three beams
was fabricated using the more common straight-legged stirrup shape, shown in Figure 4-1(b). The second series of beams also included a flexural strengthening technique using CFRP strips bonded to the underside of the test beams. One beam from each series, Series B and Series S, was tested as a control beam without the application of CFRP sheets on the web of the girders.

Two different types of bond specimens were tested in a correlative experimental program conducted by Mr. David Donald and forming part of his Master’s of Science Thesis. A total of six Rectangular Tension-type Bond Specimens were tested to evaluate the bond characteristics, as well as nine Single-Flanged Tension-type Bond Specimens designed to simulate the bottom tension flange of a typical I-shaped AASHTO bridge girder.

This chapter provides details of the specimen fabrication and specimen parameters, the test set-up and the instrumentation used to monitor the behaviour during the tests.

4.2 MATERIALS

4.2.1 CFRP Sheet Systems

Three different types of CFRP sheet systems were used for shear strengthening of the test beams. Each system consists of dry fibre sheets which are delivered to the user in rolls of continuous unidirectional fibres and a two-part epoxy resin which is mixed and applied to
the fibres on site to form the composite CFRP material. The carbon fibres are characterized by a very high tensile strength, a linearly elastic stress-strain relationship up to failure, and a modulus of elasticity slightly higher than that of steel. By comparison, the epoxy resins typically have a tensile strength and tensile modulus significantly lower than the carbon fibres, in the range of 1% to 2% of the values reported for the carbon fibres.

When bonded together and loaded in the direction of the fibres, the fibres and the resin undergo equivalent strains and the more rigid and stronger fibres carry almost all of the applied force. Since the fibres resist most of the applied load, increasing the amount of resin applied to a single-ply sheet of fibres does not significantly increase the tensile force required to rupture the composite material. However, increasing the amount of resin will significantly increase the thickness of the composite material. If the tensile force required to rupture the composite material is reported in terms of a force per unit area, it is important that the corresponding thickness of the composite material also be reported.

The thickness of the cross section corresponding to the reported tensile strength and tensile modulus of the material is commonly called the “design thickness.” Because the fibres carry most of the applied force and the amount of resin applied on site may vary widely, another material property that is useful for a true comparison of different CFRP sheet systems is the weight of the fibres per length and width of sheet. As design codes develop and terminology is standardized, the trend is moving toward reporting the
strength, modulus and thickness in terms of equivalent dry fibre properties, so that the material properties reported are independent of the amount of resin applied.

The material properties for each of the different CFRP sheet systems used in this experimental program are given in Table 4-1, as reported by the manufacturers. Both ends of one of the Series B beams, all of the Series S beams, and all of the bond specimens were strengthened using the Type B CFRP sheets. The Type B sheets are manufactured by the Mitsubishi Chemical Corporation of Japan and are known by the trade name Replark20™. The Type B sheets are similar to a sheet of paper with a design thickness of only 0.11 mm. Another beam in Series B was strengthened on both ends using the Type A CFRP sheets, which are manufactured by the Tonen Corporation of Japan and have material properties similar to those of the Type B sheets. The third type of CFRP sheets used to strengthen both ends of the final Series B beam are thicker, with a design thickness of 0.79 mm, and were handled more like a fabric than a sheet of paper. The Type C sheets are known by the trade name Tyfo™ S Fibrwrap™ and are manufactured by the Fyfe Company of California.

4.2.2 CFRP Strips

The CFRP strips used for flexural strengthening of the test beams differ from the CFRP sheet systems in that the continuous unidirectional fibres are impregnated with a polymer resin by the manufacturer and delivered to the site as a composite material. The Sika® Carbodur® CFRP strips that were used, were 1.2 mm thick by 50 mm wide, with a tensile
strength of more than 2400 MPa, an elastic modulus of 150 GPa, and a rupture strain of 19 millistrain, as reported by the manufacturer. The CFRP strips were bonded to the concrete surface using the Sikadur® 30 high-modulus, high-strength, structural epoxy paste adhesive.

4.2.3 Steel Reinforcement

Seven wire steel strand with a diameter of 13 mm was used for both the prestressed and non-prestressed flexural reinforcement. The ultimate tensile strength of the steel strand is 1860 MPa and the modulus of elasticity was 207 GPa, as reported by the precast fabricator.

The stirrups used in the Series B beams were undeformed 5.5 mm diameter steel bars. The bars were tested at the University of Manitoba and found to have an average yield strain of 2.8 millistrain, an average yield stress of 640 MPa, an average ultimate tensile strength of 683 MPa, and an average modulus of elasticity of 225 GPa. Deformed steel bars with a diameter of 6.2 mm were used to fabricate the stirrups for the Series S beams, and were included in several bond specimens to simulate the stirrups crossing the crack. The deformed bars were tested and found to have an average yield strain of 2.5 millistrain, an average yield stress of 490 MPa, an average ultimate tensile strength of 635 MPa, and an average modulus of elasticity of 191 GPa. The stress-strain relationship of the steel bars used to fabricate the straight-legged stirrups for the Series S beams is shown in Figure 4-2.
4.2.4 Concrete

The test beams were fabricated by Lafarge Canada Inc., Winnipeg, Manitoba, Canada. The specified concrete mix contained 450 kg/m$^3$ of Type 10 cement with a maximum water/cement ratio of 0.45 and a maximum aggregate size of 10 mm. Three cylinders were cast for each beam and were tested at the time of beam testing. The compressive strengths of the concrete at the time of testing ranged from 44 to 55 MPa for the beams of Series B and from 50 to 59 MPa for the Series S beams. Additional cylinders were cast by the fabricator to determine concrete strengths at the time of release of the prestressing strands, at 7 days and at 28 days.

The bond specimens were cast in the W.R. McQuade Laboratories at the University of Manitoba, with concrete provided by a local ready-mix supplier. The concrete used for the Rectangular Bond Specimens had an average compressive strength of 52 MPa, while the average compressive strength of the concrete used for the Single-Flanged Bond Specimens was 41 MPa.
4.3 BEAM TEST SPECIMENS

4.3.1 Design and Fabrication of the Test Beams

The ten meter long test beams were 1:3.5 scale models of the I-shaped Maryland Bridge girders. The beams had depth of 415 mm with a top slab of 480 mm wide and 60 mm deep as shown in Figures 4-3(a) and 4-3(b). Figure 4-4(a) shows the beam prior to casting of the concrete, while Figure 4-4(b) shows the beam after casting of the top slab. The slabs were cast a minimum of seven days after the casting of each beam. All of the beams were pretensioned with three 13 mm straight steel 7-wire strands and one draped strand. The strands were prestressed with an initial force of 100 kN, which was 55 % of their ultimate capacity. To increase the flexural capacity of the beams and avoid premature failure due to flexure, non-prestressed 13 mm steel strands were also provided. The beams of Series B were reinforced with two non-prestressed strands while three non-prestressed strands were provided in the Series S beams.

The beams of Series B were designed to carry the same shear stress at ultimate as the girders of the Maryland Bridge. The stirrup shape for the four Series B beams is shown in Figures 4-1(a) and 4-3(a), and is identical to those used in the bridge girders, with the overall dimensions and bar diameter scaled down accordingly. The straight-legged stirrup shape used for the three Series S beams, is shown in Figures 4-1(b) and 4-3(b). The spacing of the stirrups was identical in all of the test beams.
4.3.2 Specimen Parameters: Series B

The four Series B beams were fabricated using stirrups with a bent-legged shape identical to that used for the stirrups of the Maryland Bridge girders. One of the beams was tested as a control beam while the remaining three beams were strengthened using three different types of CFRP sheet systems for six different shear strengthening configurations. The beams were tested to failure at each end to determine the most efficient strengthening scheme. The specimen parameters for the eight Series B tests are summarized in Table 4-2 and in Figure 4-5 (a).

During testing of the first end of the Series B control beam, premature failure occurred due to the shape of the stirrups. An outward force was observed by spalling of the concrete cover as shown in Figure 4-6(a). This force is the resultant of the tensile forces in the vertical and diagonal legs of the stirrups and causes the stirrup to straighten. To control this outward force, the second end of the control beam in Series B was strengthened using the clamping scheme shown in Figure 4-6(b). Steel hollow structural sections (HSS) were placed on each side of the lower part of the thin web and clamped to the beam with bolts through the web.

The second Series B beam was strengthened using one layer of 250 mm wide vertical CFRP sheets with a 100 mm gap between each sheet to allow drainage of any moisture accumulation. The vertical CFRP sheets were applied on each side of the cross section, as shown in Figure 4-6 (c), from the top of the beam immediately below the slab to the
underside of the beam where they were overlapped for a minimum length of 100 mm.
The other end of the beam was strengthened with a single layer sheet of 220 mm wide
CFRP, with the continuous carbon fibres in the horizontal direction, on top of the vertical
sheets as shown in Figure 4-7. The surface of the second beam was prepared prior to
application of the CFRP sheets using a grinder, wire brush and high-pressure air for
cleaning the surface after grinding. Type A CFRP sheets were used to strengthen both
ends of the second test beam.

One end of the third test beam was strengthened with a single layer of 250 mm wide
CFRP sheets with the fibres oriented diagonally at 45 degrees. The sheets were applied
on each side of the beam and overlapped on the underside of the beam. A 20 mm gap
was provided between each diagonal sheet as shown in Figure 4-8. The other end of the
third beam was strengthened using one layer of 250 mm wide vertical CFRP sheets
similar to the second beam, but with a gap of 20 mm between the sheets. Type B CFRP
sheets were used for both ends of the third beam. The surface of the beam was prepared
using a high-pressure water-blasting technique, however, some grinding was required to
round any sharp corners on the beam.

The fourth test beam was strengthened using one layer of 250 mm wide diagonal CFRP
sheets. As with the previous beams, the sheets were applied on each side and overlapped
on the underside of the beam. A 100 mm gap was provided between each diagonal sheet
similar to the second test beam. The other end of the beam was strengthened with a
single layer sheet of 220 mm wide CFRP, with the continuous carbon fibres in the
horizontal direction, on top of the diagonal sheets as shown in Figure 4-9. The fourth beam was strengthened using Type C CFRP sheets. Similar to the third test beam, the surface of the beam was prepared using the hydro-blasting technique, and grinding to round any sharp corners on the beam.

4.3.3 Specimen Parameters: Series S

The three Series S beams were fabricated using the more common straight-legged stirrup shape, and with a slightly larger diameter deformed steel bar than was used for Series B. The first end of the first Series S beam was tested to failure without any strengthening. A flexural strengthening technique using CFRP strips bonded to the underside of the beams was then applied to the second end of the first Series S beam. This beam with the CFRP strips, but without any shear strengthening scheme, was then tested to failure as a control beam. The remaining two beams were strengthened using both the CFRP strips and the CFRP sheets, and were tested to failure at each end. The specimen parameters for the six Series S tests are summarized in Table 4-3 and in Figure 4-5 (b).

Type B CFRP sheets were used on top of the CFRP strips for both of the remaining Series S beams. Both of the beams were strengthened for shear on both ends using a single layer of 250 mm wide CFRP sheets with the fibres oriented diagonally at 45 degrees. The sheets were applied on each side of the cross section, and were overlapped on top of the CFRP strip on the underside of the beam. A 20 mm gap was provided between each diagonal sheet. Prior to application of the sheets, the surface of both of the...
beams was prepared using the hydro-blasting technique.

The first end of the second Series S beam was tested to failure with a single layer of diagonal CFRP sheets. On the second end of the same beam, a single layer 220 mm wide horizontal CFRP sheet was applied on top of the diagonal sheets as shown in Figure 4-10. The first end of the third Series S beam was strengthened with a second layer of diagonal sheets directly on top of the first layer. A clamping scheme similar to that used on the Series B beam was applied on top of the single layer of diagonal sheets on the second end of the final Series S beam, as shown in Figure 4-11. This clamping scheme was applied to control the outward force that develops due to the shape of the beam cross section and the increasing tensile force in the CFRP sheets.

4.3.4 Application of the External Bonded CFRP Systems

The CFRP strips used for flexural strengthening were applied to the underside of the Series S beams by local a contractor and using the procedure recommended by strip the manufacturer. After the concrete surface was cleaned and prepared, the Sikadur® epoxy paste adhesive was applied to the concrete substrate and to one side of the Carbodur® CFRP strips, as shown in Figure 4-12(a). The strips were then pressed into place, as shown in Figure 4-12(b), and using a rubber roller to squeeze out the excess adhesive. A second layer of Carbodur® was then applied on top of the first layer using the same procedure.
As with the CFRP strips, the CFRP sheets used for shear strengthening were applied to the beams using well-defined procedures recommended by each sheet manufacturer and proprietary products supplied by each manufacturer. Two local contractors applied the sheets, with one contractor using the Tonen sheets and the other applying the Mitsubishi and Fyfe sheets.

After the concrete surface of each beam was cleaned and prepared, an epoxy primer was applied followed by an epoxy paste and finally, the epoxy resin, which forms the matrix of the composite CFRP material. The epoxy primer is the least viscous of the three epoxy components, penetrating and sealing the porous surface of the concrete to improve the bond between the concrete and the epoxy resin matrix. After setting of the primer, any significant surface irregularities were filled using a thick epoxy paste, as shown in Figures 4-13(a) and 4-13(b). Once a smooth, flat surface was achieved, the epoxy impregnation resin and the carbon fibre sheets were applied to form the composite CFRP material. The viscosity of the epoxy resin was low enough to allow the resin to impregnate the dry fibre sheets, but high enough to keep the sheets attached to the beam while the resin sets and cures. Figures 4-14(a), 4-14(b) and 4-14(c), show the application of the Type A - Tonen, Type B - Mitsubishi and Type C – Fyfe sheets, respectively.

All three of the CFRP sheet systems were applied using the same general procedure, however, there were some differences specific to each system as described below. For the beam strengthened using the Fyfe system, the surface irregularities were filled using the epoxy impregnation resin with additional filler material, added on site, to thicken the
resin to the consistency of a paste. The Mitsubishi and Tonen epoxy pastes were provided separately from the epoxy resins and did not require any additional filler material. For the beams strengthened using the Mitsubishi and Tonen systems, additional grinding was required, after setting of the epoxy paste, to achieve a smooth, flat surface.

For the beams strengthened with the Mitsubishi and Tonen systems, the epoxy resin was applied to the surface of the beam followed by the CFRP sheets. A second layer of epoxy resin was applied as a top coat. In the case of multiple layers of sheets, the top coat of epoxy resin served as a base coat for the next layer of CFRP sheets. By comparison, the Fyfe dry fibre sheets were impregnated with epoxy resin in a separate resin bath prior to application on the beam, as shown in Figure 4-15. Any subsequent layers were applied using the same technique.

4.5 BEAM TEST SET-UP

Testing was conducted at the W.R. McQuade Laboratories, University of Manitoba. As shown in Figures 4-16(a) and 4-16(b), the simply supported beams were subjected to two equivalent non-symmetric point loads spaced according to a typical highway truck and the scale of the specimens. The monotonic static load was applied using an MTS 5000 kN testing machine under stroke control. The shear span of 1940 mm was kept constant for all of the tests, while the overall span was varied.
The first end of each Series B beam was tested with an overall span of 9.7 m, as shown in Figure 4-16(a). In order to test the second end of each Series B beam, the beam was turned and one support was moved to exclude the damaged first end of the beam, as shown in Figure 4-16(b). The overall span for testing of the second end of the Series B beams was 6.05 m. Both ends of all of the Series S beams were tested with the same overall span of 6.05 m, using the test set-up shown in Figure 4-16 (b).

The beams were simply supported on rollers, which rested on concrete blocks. The load was applied through steel beams on the full width of the top slab. Plaster was used to distribute the load evenly at the loading points and at the supports. Lateral support was provided, without restricting the vertical displacement of the specimen, at the location of maximum moment and at the supports. Additional lateral support was provided for the longer 9.7 m spans.

3.6 BEAM INSTRUMENTATION

The strain in the stirrups and the distribution of the strain in the CFRP sheets were monitored using electrical resistance strain gauges with a 5 mm gauge length. DEMEC stations were used to measure flexural strains at the top and bottom of the beam. The strain in the prestressed and non-prestressed strands was monitored using electrical resistance strain gauges. Displacement gauges (PI gauges) with 100 mm gauge lengths were used to measure the compressive strain at ultimate on the top concrete surface of the
slab. For some beams, displacement gauges and DEMEC stations were used to form rosettes for strain measurement in three directions on the web. Deflection was measured using Linear Voltage Displacement Transducers (LVDT) at each of the load points.

Because the strain in the CFRP sheets can be very localized, the strain gauges were applied very close together along the length of the continuous carbon fibres to monitor the distribution of strain along a particular group of fibres. In general, one column of gauges was applied 50 mm from each edge of the 250 mm wide CFRP sheets within the shear span. For most of the beams, a maximum spacing of 30 to 40 mm was used between the strain gauges in each column. The location and number of strain gauges applied to the CFRP sheets for the beam shown in Figure 4-17 is typical for all of the test beams strengthened with CFRP sheets.

During each test, displacement and strain readings were recorded in parallel, using data acquisition systems, at a rate of 1 sample every 2 seconds. The load and stroke of the machine cross-head were also recorded at a rate of 1 sample every 2 seconds. Two data acquisition systems with a total capacity of 72 channels were used for the Series B beams while one data acquisition system with 29 channels was used for the Series S beams. For the Series S beams, strain gauge readings were also taken manually using a strain indicator box to monitor a total of 40 channels.
4.7 BOND SPECIMENS

The Rectangular and Single-Flanged Bond Specimens consisted of reinforced concrete prisms strengthened on opposite faces with 200 mm wide Type B CFRP sheets and subjected to uniaxial tension as shown in Figures 4-18 (a) and (b). For all of the bond specimens, the CFRP sheets were applied with the continuous longitudinal fibres oriented parallel to the direction of the applied tensile load.

Due to the arrangement of the internal reinforcing, cracking of the concrete was initiated at mid-height of the specimen as shown in Figures 4-18 (a) and (b). Two different crack angles were used, with the angle of the crack at 45 degrees, or perpendicular to the longitudinal fibres at zero degrees. For some specimens, steel reinforcement crossing the crack was used to simulate the effect of stirrups, in order to evaluate load sharing between the CFRP sheets and the stirrups. The same type of steel bars that were used to fabricate stirrups for the Series S beams, were used to simulate stirrups in the bond specimens. The CFRP sheets were applied to the bond specimens using the same procedures and the same two surface preparation techniques as described previously for the test beams.

A total of six Rectangular Bond Specimens were fabricated and tested with dimensions 100 x 275 x 900 mm. Table 4-4 provides a summary of the specimen parameters for the Rectangular Bond Specimens.
Nine 275 x 900 mm Single-Flanged Bond Specimens were fabricated and tested to evaluate the effect of the concrete surface profile which simulates the lower tension flange of an I-shaped AASHTO bridge girder, as shown in Figure 4-18 (b). In addition to varying the crack angle, the distance between the crack and the interior corner of the flanged section was varied between 0 and 50 mm. Table 4-5 provides a summary of the parameters for the Single-Flanged Bond Specimens.

The bond specimens were tested under monotonic loading and stroke control. As shown in Figure 4-19, pin-ended connections and rotating couples were used to ensure an even distribution of force in the two tension bars protruding from each end of the specimen. Closely spaced 5 mm strain gauges were applied on both sides of the specimens, to measure the distribution of axial strain along the length and across the width of the sheets, and the length of sheet over which force is effectively transferred to the concrete. U-shaped displacement gauges (PI gauges) with 100 mm and 200 mm gauge lengths were also used to monitor strain across the crack on both faces and both sides of the bond specimens. Any potential eccentricity in the specimen was monitored using a comparison of the displacement gauge measurements.
### Table 4-1 Material Properties of CFRP Sheets

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<th>Type B*</th>
<th>Type C*</th>
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<td>0.79</td>
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<td>Tensile Modulus (GPa):</td>
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*properties for dry fiber sheets, *properties for composite fiber and resin sheets

### Table 4-2 Specimen Parameters: Series B

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<th>Layer 2 Config.</th>
<th>gap (mm)</th>
<th>s_t (mm)</th>
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<th>Span (m)</th>
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### Table 4-3 Specimen Parameters: Series S

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<td>Clamped</td>
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<td>370</td>
<td>B</td>
<td>59</td>
<td>6.0</td>
<td>S-Diag-CL</td>
<td></td>
</tr>
</tbody>
</table>
### Table 4-4 Parameters for Rectangular Bond Specimens

<table>
<thead>
<tr>
<th>Surface Preparation</th>
<th>Steel at Crack</th>
<th>Crack Angle</th>
<th>Specimen Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>grinding</td>
<td>no</td>
<td>0</td>
<td>R-0-G</td>
</tr>
<tr>
<td></td>
<td>no</td>
<td>45</td>
<td>R-45-G</td>
</tr>
<tr>
<td>hydro-blasting</td>
<td>no</td>
<td>0</td>
<td>R-0-H</td>
</tr>
<tr>
<td></td>
<td>no</td>
<td>45</td>
<td>R-45-H</td>
</tr>
<tr>
<td></td>
<td>yes</td>
<td>0</td>
<td>R-0-H-S</td>
</tr>
<tr>
<td></td>
<td>yes</td>
<td>45</td>
<td>R-45-H-S</td>
</tr>
</tbody>
</table>

### Table 4-5 Parameters for Single-Flanged Bond Specimens

<table>
<thead>
<tr>
<th>Crack Angle</th>
<th>Distance to Crack*</th>
<th>Steel at Crack</th>
<th>Control of Peeling</th>
<th>Specimen Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 degrees</td>
<td>0 mm</td>
<td>no</td>
<td>none</td>
<td>SF-0-0</td>
</tr>
<tr>
<td>0 degrees</td>
<td>50 mm</td>
<td>no</td>
<td>none</td>
<td>SF-0-50</td>
</tr>
<tr>
<td>30 degrees</td>
<td>0 mm</td>
<td>no</td>
<td>none</td>
<td>SF-30-0</td>
</tr>
<tr>
<td>30 degrees</td>
<td>50 mm</td>
<td>no</td>
<td>none</td>
<td>SF-30-50</td>
</tr>
<tr>
<td>30 degrees</td>
<td>0 mm</td>
<td>yes</td>
<td>none</td>
<td>SF-30-0-S</td>
</tr>
<tr>
<td>0 degrees</td>
<td>50 mm</td>
<td>yes</td>
<td>none</td>
<td>SF-0-50-S</td>
</tr>
<tr>
<td>0 degrees</td>
<td>50 mm</td>
<td>no</td>
<td>wrapped</td>
<td>SF-0-50-W</td>
</tr>
<tr>
<td>0 degrees</td>
<td>50 mm</td>
<td>no</td>
<td>horizontal</td>
<td>SF-0-50-H</td>
</tr>
<tr>
<td>0 degrees</td>
<td>50 mm</td>
<td>no</td>
<td>clamped</td>
<td>SF-0-50-C</td>
</tr>
</tbody>
</table>

* *Shortest Distance from Interior Angle to Crack*
Figure 4-1(a) Bent-Legged Stirrup for Series B Beams

Figure 4-1(b) Straight-Legged Stirrup for Series S Beams

Figure 4-2 Strain vs Applied Force: Steel Bars for Stirrups in Series S
Figure 4-3(a) Test Beam Cross Section: Series B

Figure 4-3(b) Test Beam Cross Section: Series S
Figure 4-4 (a) Test Beam Prior to Casting of Concrete

Figure 4-4 (b) Test Beam After Casting of Top Slab
Figure 4-5 (a) Summary of Series B Beams

Series S Beams – all Type B CFRP Sheets

Figure 4-5 (b) Summary of Series S Beams
Figure 4-6(a)  Premature Stirrup Failure: Series B Control Beam

Figure 4-6(b)  Stirrup Clamping Scheme

Figure 4-6 (c)  CFRP Sheets on Each Side of Cross Section
Figure 4-7  Beam with Horizontal and Vertical Sheets: B-Vert-H

Figure 4-8  Beam with Diagonal Sheets and 20 mm Gap: B-Diag20

Figure 4-9  Beam with Horizontal and Diagonal Sheets: B-Diag-H
Figure 4-10: Beam with Diagonal and Horizontal Sheets: S-Diag-H

Figure 4-11: Beam with Clamped Diagonal Sheets: S-Diag-CL
Figure 4-12 (a) Application of the Adhesive to the CFRP strips

Figure 4-12 (b) Application of the CFRP Strips onto Series S Beam
Figure 4-13 (a) Application of the Epoxy Paste: Type A (Tonon) System

Figure 4-13 (b) Application of the Epoxy Paste: Type C (Fyfe) System
Figure 4-14 (a) Application of the Type A (Tonen) CFRP Sheets

Figure 4-14 (b) Application of the Type B (Mitsubishi) CFRP Sheets
Figure 4-14 (c) Application of the Type C (Fyfe) CFRP Sheets

Figure 4-15 Separate Resin Bath for Wet Lay-up of Type C (Fyfe) CFRP Sheets
Figure 4-16 (a) Test Set-Up: End 1

Figure 4-16 (b) Test Set-Up: End 2

Figure 4-17 Typical Strain Gauge Locations on CFRP Sheets
0° OR 45°

Internal Steel Crossing the Crack

*Figure 4-18 (a) Rectangular Bond Specimen*
Figure 4-18 (b)  Single-Flanged Bond Specimen
Figure 4-19  Bond Test Set-up
EXPERIMENTAL RESULTS AND ANALYSIS

5.1 GENERAL

Seven I-shaped prestressed concrete girders were strengthened in shear, using three different types of CFRP sheets for ten different sheet configurations. The ten meter long beams are 1:3.5 scale models of the girders used for the Maryland Bridge in Winnipeg, Manitoba, Canada. The first series of four beams, Series B, were fabricated using internal shear reinforcement with a bent-legged shape identical to that used in the Maryland Bridge. In order to extend the applicability of the experimental results to typical AAHSTO girders, the second series of three beams, Series S, were fabricated using the more common straight-legged stirrup shape. The Series S beams also included a flexural strengthening technique using CFRP strips bonded to the underside of the test beams. The beams were tested to failure at each end to examine their behaviour and determine the efficiency of each strengthening scheme. One beam from each series was tested as a control beam without the application of CFRP sheets on the web of the girders. This chapter describes the results of the fourteen beam tests conducted in this experimental program.
Tables 5-1 and 5-2 provide a summary of the parameters evaluated and a comparison of the ultimate shear failure loads, $V_u$, for all of the beams in Series B and Series S, respectively. Since the shape of the stirrups plays a key role in the behaviour of the beams, the test results for each series of beams are discussed separately, with comparisons between series made only where applicable.

For all of the beams tested in this experimental program, the bond between the CFRP sheets and the concrete was a critical factor controlling the shear capacity of the beam. In general, shear-tension failure within the concrete substrate controlled the overall beam failure in beams strengthened with CFRP sheets, rather than rupture of the sheets or debonding between the sheets and the concrete. Therefore, in addition to the beam tests, fifteen bond specimens were also tested in order to characterize the bond between the concrete and the CFRP sheets used in this strengthening method. Load sharing between the external CFRP sheets and the internal steel was also examined in four of the bond specimens.

Two different types of bond specimens were tested, and test results for the six Rectangular Bond Specimens and nine Single-Flanged Bond Specimens are discussed in this chapter. In the final two sections of this chapter, the beam test results and the bond test results are correlated through an analysis and discussion of the strain distribution and maximum strain in the CFRP sheets, as well as the load sharing between the CFRP sheets and the steel stirrups.
5.2 GENERAL BEHAVIOUR: SERIES B BEAMS

In all of the Series B beam tests, flexural shear cracks were observed within the shear span and extended toward the top flange at ultimate. For the beams strengthened with externally bonded CFRP sheets, failure within the concrete substrate was generally observed, rather than debonding between the CFRP sheets and the concrete. The behaviour during testing and mode of failure for each Series B beam is discussed in more detail in the following sub-sections.

5.2.1 Beam B-Control

The unstrengthened end of the first Series B beam was tested to failure as a control beam, and reached an ultimate shear capacity, $V_u$, of 137 kN. Figure 5-1(a) shows beam B-Control at failure. Just prior to failure, spalling of the concrete cover was observed due to an outward tensile force resultant causing straightening of the bent corner of the stirrups. Figure 5-1(b) illustrates the straightening behaviour of the stirrups in the Series B control beam.

Due to the bent-legged shape of the stirrups, only one of the stirrups in the Series B control beam reached yield before premature failure occurred due to spalling of the concrete cover and straightening of the stirrups. Figure 5-2 illustrates the location of the stirrups, the location of concrete spalling, and the crack pattern observed just prior to failure.
5.2.2 Beam with Clamped Bent-Legged Stirrups: B-CL

The clamping scheme applied to beam B-CL was effective in controlling the outward force in the stirrups. No spalling of the concrete cover was observed, as illustrated by the behaviour of beam B-CL at failure shown in Figure 5-3. Because premature failure due to straightening of the bent stirrup legs was prevented, all of the measured stirrup strains exceeded the yield strain of 2.8 millistrain at failure, and the distribution of forces between the clamped stirrups was improved. Figure 5-4 shows the location of the stirrups and the crack pattern observed just prior to failure. A comparison of Figure 5-2 and Figure 5-4 suggests that the crack pattern on the web is similar for both beams B-Control and B-CL.

The maximum strains in the clamped bent-legged stirrups were all significantly higher than those observed for the same stirrups in beam B-Control. Figure 5-5 shows the stirrup strain versus applied shear load curves for beams B-CL and B-Control. The increased ductility and improved distribution of forces among the clamped stirrups contributed to the observed 27% increase in the ultimate shear capacity, $V_u$, when compared to beam B-Control.

Figure 5-5 also indicates that for a given level of applied shear load less than ultimate, the stirrup clamping scheme did not reduce the tensile force in the stirrups when compared with beam B-Control. However, the clamping scheme did enable all of the bent-legged stirrups to reach the design yield stress, without failing prematurely due to their shape.
5.2.3 Beams with Vertical CFRP Sheets: B-Vert20 and B-Vert100

Two different types of CFRP sheets, Types A and B, with similar thickness and material properties were used for the beams strengthened with vertical sheets. Two gap sizes and surface preparation techniques were also used as shown in Table 5-1. The ultimate shear capacity, $V_u$, of beam B-Vert20 increased by 17% compared with beam B-Control, while only a 10% increase was observed for beam B-Vert100. This behaviour was expected, since the reduced gap size for B-Vert20 results in a 32% increase in the area of sheets, per unit length of beam, when compared to beam B-Vert100.

Figures 5-6 and 5-7 show the beams with vertical CFRP sheets, at failure. In both beams above and below the shear cracks, shear failure in the concrete substrate was generally observed, with only minor areas of localized debonding at the sheet-to-concrete interface occurring in each beam. For beam B-Vert20, with the surface prepared by hydro-blasting, the CFRP sheets ruptured in some locations, as can be observed in Figure 5-6. Figure 5-7 (b) shows the CFRP sheets peeled back following failure of beam B-Vert100, for which grinding was used as a surface preparation technique. As illustrated in Figure 5-7 (b), concrete remained bonded to the CFRP sheet at failure, indicating that the predominant mode of failure for the CFRP sheets was shear failure within the concrete substrate. It should be noted that the sheets were peeled back following failure, and most of the area without concrete attached was debonded at this time, not during testing. The extent to which the sheets were manually peeled and debonded from the beam is evident in a comparison of Figures 5-7 (a) and 5-7 (b).
5.2.4 Beams with Diagonal Sheets: B-Diag20 and B-Diag100

The thickness and material properties of the Type B and Type C CFRP sheets, which were used for the diagonal configurations in Series B, vary significantly as shown in Table 4-1. The gap size parameter was also varied for beams B-Diag20 and B-Diag100, as shown in Table 5-1, while the surface preparation technique was not. For both beams, similar 26% and 29% increases in ultimate shear capacity, $V_u$, were achieved when compared with the Series B control beam.

Due to the shape of the girder, straightening of the CFRP sheets was observed on both beams at the bottom of the web prior to failure, as shown in Figure 5-8. Figures 5-9 and 5-10 show beams B-Diag100 and B-Diag20, respectively, at failure. After each test, the CFRP sheets were removed and extensive failure in the concrete substrate was observed.

The efficiency of the diagonal CFRP sheets is evident when comparing the stirrup strain versus applied shear load curves provided in Figure 5-11. The stirrup strain at any level of applied shear is lower for the beams with diagonal sheets. Although the beam with horizontal and vertical sheets reached a higher ultimate shear load, the stirrup strain was greater. Figure 5-11 also suggests that, in spite of the larger gap size, beam B-Diag100 exhibited lower stirrup strains than beam B-Diag20, and therefore a greater contribution from the thicker Type C CFRP sheets, at the same level of applied shear.
5.2.5 Beams with Horizontal Sheet Combinations: B-Vert-H and B-Diag-H

Beams B-Vert-H and B-Diag-H, with a single layer horizontal CFRP sheet applied on top of vertical or diagonal sheets, achieved similar 34 % and 36 % increases in ultimate shear capacity, \( V_u \), when compared with beam B-Control. In both beams at ultimate, failure in the concrete substrate was observed as well as some rupture of the CFRP sheets very close to the top of the web, as illustrated in Figures 5-12 (a) and 5-13 (a). Following each test, the CFRP sheets were removed to observe that shear failure in the concrete substrate was the predominant mode of failure for the CFRP sheets, as shown in Figures 5-12 (b) and 5-13 (b).

Similar to the beams with vertical CFRP sheets only, beam B-Vert-H demonstrated spalling at the bottom of the web, due to the outward force in the stirrups at this location. However, due to the presence of both horizontal and vertical sheets, spalling was observed at a higher level of applied shear load than in beams B-Control, B-Vert100 or B-Vert20.

Similar to the beams with diagonal CFRP sheets only, on beam B-Diag-H with the horizontal and diagonal sheet combination, some straightening of the diagonal sheets was observed at the bottom of the web. However, the observed straightening was less extensive at ultimate.
Figure 5-14 provides a comparison of the stirrup strain versus applied shear load for the Series B beams with horizontal sheets, B-Vert-H and B-Diag-H. For each of these beams, a similar beam without horizontal sheets was tested. For beams B-Vert100 and B-Vert-H with vertical sheets, the application of the horizontal sheet reduced the strain in the stirrups at the same level of applied shear load, and increased the ultimate shear capacity significantly. For beams B-Diag100 and B-Diag-H with diagonal sheets, the application of the horizontal sheet increased the ultimate shear capacity, but did not reduce the strain in the stirrups any further, as shown in Figure 5-14. The diagonal configuration remains the most efficient in reducing the level of strain in the stirrups at any given level of applied shear. Reduced stirrup strain would be desirable where the existing stirrups have a bent-legged shape and the potential to straighten under tension.

5.3 GENERAL BEHAVIOUR: SERIES S BEAMS

Similar to the Series B test results, all of the Series S beams failed in shear. For the beams strengthened with externally bonded CFRP sheets or strips, failure within the concrete substrate was typically observed. The behaviour during testing and mode of failure for each Series S beam is discussed in more detail in the following sections.

5.3.1 Unstrengthened Beam: S-NoFRP

Since most of the Series S beams were strengthened in flexure using CFRP strips, for
comparison purposes, one unstrengthened beam, S-NoFRP, was tested to failure without applying the CFRP strips. During testing, flexural shear cracks were observed within the shear span and extended toward the top flange at ultimate. Figure 5-15 shows beam S-NoFRP at failure.

The ultimate shear capacity, \( V_u \), of beam S-NoFRP was 206 kN, as shown in Table 5-2. When compared with beam B-Control, reinforced with bent-legged stirrups, an increase in the ultimate shear capacity of 50% was observed. This significant increase is partly due to the premature failure of beam B-Control caused by straightening of the bent stirrup legs. A comparison of beam S-NoFRP and beam B-CL with the clamped bent-legged stirrups, shows an 18% increase in the ultimate shear capacity. The increased shear capacity can be attributed to the larger area of reinforcing bar used for the straight-legged stirrups.

5.3.2 Beam S-Control

Beam S-Control was strengthened for flexure, using CFRP strips bonded to the underside of the beam, and tested to failure as a control beam for the Series S beams. During testing, at an applied shear load of 213 kN, shear failure in the concrete substrate above the CFRP strip occurred, and the load dropped to 182 kN. Failure in the concrete substrate was initiated by relative vertical displacement on either side of a shear crack occurring near the end block of the beam, and propagated all along the length of the CFRP strip. A layer of concrete remained bonded to the strip as shown in Figure 5-16,
indicating that surface shear failure within the concrete substrate had occurred. Debonding between the first and second layers of CFRP strips was also observed over much of the beam. The failure in the concrete substrate occurred suddenly and in a zipper-like fashion. This type of failure is well documented in the literature, and is considered a premature failure of this flexural strengthening technique.

Since the beam had not yet reached its unstrengthened flexural capacity when premature failure of the flexural strengthening system occurred, the beam continued to carry load until beam shear failure occurred at an applied shear load of 196 kN. Figure 5-17 shows the ultimate shear failure of beam S-Control. As shown in Table 5-2, there was no significant increase in the shear capacity of beam S-Control when compared with the unstrengthened beam without the CFRP strip, S-NoFRP.

5.3.3 Beams with Diagonal Sheets: S-Diag-1 and S-Diag-2

Beam S-Diag-1, with a single layer of diagonal CFRP sheets, reached an ultimate shear capacity of 233 kN. A second layer of diagonal sheets, applied to beam S-Diag-2, did not increase the shear capacity significantly and the beam failed at 234 kN.

Similar to the beams with diagonal sheets tested in Series B, both Series S beams exhibited straightening of the diagonal sheets due to the shape of the girder. The straightening of the diagonal sheets prior to failure is shown in Figure 5-18, and was more pronounced for the beams of Series S when compared with those of Series B.
For both beams, the CFRP strips remained bonded to the underside of the beam until beam shear failure occurred. Failure in the concrete substrate above the CFRP strips was observed only in the zone of beam shear failure. The first and second layers of CFRP strips remained bonded to one another, as shown in Figure 5-19.

Figure 5-20 (a) shows beam S-Diag-1 at failure, while Figure 5-20 (b) shows beam S-Diag-2 at failure. The observed failure of these two beams was so similar that it is difficult to distinguish one photograph from the other. For both beams, concrete remained bonded to the sheets at failure, indicating that shear failure in the concrete substrate occurred, rather than debonding at the sheet-to-concrete interface. In a few locations at the top of the web, rupture of the CFRP sheets was observed.

The observed behaviour and ultimate shear capacity of the Series S beams with single and double layers of diagonal sheets were similar. However, the strains measured in the double layer of CFRP sheets were typically about one-half to two-thirds the magnitude of the strains measured in the single layer sheets, at the same locations on each beam. Therefore, the total shear resisting force in the single layer of diagonal CFRP sheets was not significantly increased with the application of a second layer of diagonal sheets. A more detailed analysis of the contribution of the single and double layer sheets to the ultimate shear capacity of each beam will be described in Chapter 6.

Both the single layer and double layer diagonal CFRP sheets were not fully effective due to the shape of the girder, and the increase in ultimate shear capacity for beams S-Diag-1
and S-Diag-2 was only 9 % and 10 %, respectively, when compared to beam S-Control.

5.3.4 Beam with Diagonal and Horizontal Sheets: S-Diag-H

Beam S-Diag-H was strengthened with a single layer horizontal sheet on top of a single layer diagonal sheet and reached an ultimate shear capacity of 247 kN, as indicated in Table 5-2. Similar to the beams with diagonal sheets alone, the diagonal sheets on beam S-Diag-H began to straighten at the bottom of the web. However, the straightening of the diagonal sheets was not as pronounced for beam S-Diag-H prior to failure. Based on the 16 % increase in the ultimate shear capacity of beam S-Diag-H, when compared with beam S-Control, applying a horizontal layer on top of a single diagonal layer of CFRP sheets is more effective than applying a second layer of diagonal sheets.

Figures 5-21 (a) and (b), show beam S-Diag-H at failure. Similar to the beams with diagonal sheets alone, the CFRP strips remained bonded to the underside of the beam until beam shear failure occurred and the first and second layers of CFRP strips remained bonded to one another even after failure occurred. However, unlike the beams with diagonal sheets alone, debonding between the CFRP strip and the adhesive did occur on beam S-Diag-H, and extended over a longer portion of the shear span, from the zone of shear failure all the way to the support.

As shown in Figures 5-21 (a) and (b), the horizontal sheet ruptured at each end of the shear span, in locations near the applied load and near the end-block to thin-web
transition zone. Concrete remained bonded to the CFRP sheets over most of the beam at failure, indicating that shear failure in the concrete substrate occurred, rather than debonding of the CFRP sheets.

5.3.5 Beam with Clamped Diagonal Sheets: S-Diag-CL

The clamping scheme applied to beam S-Diag-CL was effective in controlling the straightening of the diagonal CFRP sheets. The sheets remained completely bonded to the beam and failure occurred outside of the strengthened zone, as shown in Figure 5-22.

Failure in beam S-Diag-CL occurred at an applied shear load of 272 kN. Since the beam failed outside of the strengthened zone, the full potential of the clamped diagonal sheets was not realized. The 28 % increase in ultimate shear capacity for beam S-Diag-CL, when compared to beam S-Control, could have been even higher if failure in the unstrengthened end-block zone had not occurred.

By preventing straightening of the sheets, the clamping scheme allows for the effective use of the sheets on this girder shape. The strains measured in the clamped diagonal CFRP sheets reached much higher levels than those measured for other Series S beams, as shown in Figure 5-23 (a). For the other Series-S beams without a horizontal layer or clamping, the reduction in strain with increasing load is due to the straightening of the sheets. Correspondingly, the strain in the stirrups was reduced for beam S-Diag-CL, as illustrated by the stirrup strain versus applied shear load curves in Figure 5-23 (b).
5.4 GENERAL BEHAVIOUR: BOND SPECIMENS

5.4.1 Rectangular Bond Specimens

A summary of test results for the Rectangular Bond Specimens are provided in Table 5-3. Because the strain measured in the FRP sheets along the crack typically varied slightly from one edge of the sheet to the other edge of the sheet, as shown in Figures 5-24 (a) and (b), the average ultimate strain measured at the crack, \( (\varepsilon_{f_{\text{ult}}})_{\text{meas}} \), is reported in Table 5-3. Also provided in Table 5-3, is the calculated average ultimate strain, \( (\varepsilon_{f_{\text{ult}}})_{\text{calc}} \), which is determined based on the tensile force in the FRP sheets at ultimate, \( T_{f_{\text{ult}}} \), and the area, \( A_f \), and modulus of elasticity, \( E_f \), of the FRP crossing the crack as follows:

\[
(\varepsilon_{f_{\text{ult}}})_{\text{calc}} = \frac{T_{f_{\text{ult}}}}{(A_f E_f)} \quad (5-1)
\]

For the two specimens with internal steel crossing the crack, the ultimate tensile force in the sheets, \( T_{f_{\text{ult}}} \), is determined by subtracting the force in the internal steel from the applied tensile load. For both specimens, the steel reached yield at ultimate.

As shown in Table 5-3, the calculated average ultimate strain is typically slightly higher than the measured average ultimate strain. The higher calculated strain may be due to the use of a value for the modulus of elasticity, \( E_f \), as reported by the CFRP sheet manufacturer, that is slightly lower than the actual value. In spite of the slight discrepancies in the measured and calculated values, it can be concluded that the average
ultimate strain in the CFRP sheets was in the range of 0.006 to 0.008 for the Rectangular Bond Specimens.

Two different types of failure were observed for the Rectangular Bond Specimens, depending upon the technique used for concrete surface preparation. For the specimens prepared using the grinding technique, shear-tension failure within the concrete substrate was observed. Failure in the concrete substrate first occurred near the crack, then propagated along the length of the sheet until ultimate failure of the specimen occurred, as shown in Figures 5-25 (a) and (b). For the specimens prepared using the hydro-blasting technique, failure occurred due to rupture of the CFRP sheets at a higher strain level than was observed for the ground specimens. Figures 5-26 (a) and (b) show two hydro-blasted rectangular bond specimens at ultimate. The use of hydro-blasting improved the bond and increased the tensile capacity by approximately 9%.

5.4.2 Single-Flanged Bond Specimens

As anticipated, the Single-Flanged Bond Specimens failed due to peeling and straightening of the CFRP sheets at the interior corner, as shown in Figure 5-27. A summary of the test results for the Single-Flanged Bond Specimens are provided in Table 5-4.

The average ultimate strain in the CFRP sheets was lower for the Single-Flanged specimens with the crack located directly at the interior corner when compared to
specimens with the crack located 50 mm from the interior corner. Similar to the Rectangular specimens, the average ultimate strain was higher for specimens with steel crossing the crack, when compared to those with the same parameters, but without steel.

In general, the average ultimate strain in the Single-Flanged specimens was in the range of 0.004 to 0.006, with the exception of the specimen with the clamped sheets and specimen SF-0-50-S.

5.5 STRAIN DISTRIBUTION IN CFRP SHEETS

The bond between the CFRP sheets and the concrete is a critical factor controlling the capacity of most beams strengthened in shear using externally bonded CFRP sheets. In order to characterize the bond mechanism and load transfer between the CFRP sheets and the concrete, the distribution of strain along the principal fibres of the CFRP sheets is examined first for the bond specimens and then for the more complex beam specimens. The strain in the principal fibres of the CFRP sheets is also examined along the shear cracks in the test beams, since this strain distribution is directly related to the overall shear resistance provided by the CFRP sheets. The effect of the type and configuration of CFRP sheets on the strain distribution along the shear crack is discussed, as well as the effect of the CFRP sheet stiffness and the shape of the beam cross section on the maximum strain that can be developed in the CFRP sheets. A model is introduced to predict the maximum strain developed in CFRP sheets bonded to concrete and the
effective bond length over which this load transfer takes place.

5.5.1 Strain Distribution Along Principal Fibres: Bond Specimens

For each of the bond specimens, several closely spaced 5mm strain gauges were applied along the principal fibres of the CFRP sheets to examine the strain distribution during testing, as shown in Figures 5-28 and 5-29. The strain distributions for the three Rectangular Bond Specimens with the crack perpendicular to the longitudinal fibres, R-0-G, R-0-H, and R-0-SH, are shown in Figures 5-28 (a), (b), and (c), respectively. Figures 5-29 (a), (b) and (c) show the strain distributions for the three Rectangular Bond Specimens with the 45 degree crack angle, R-45-G, R-45-H, and R-45-H-S, respectively.

For all six specimens, the strain distribution in the CFRP sheets is characterized by a zone of rapid strain increase or load transfer, as well as a zone of relatively constant strain closer to the crack where load transfer is no longer taking place. As shown in Figures 5-28 (a) and 5-29 (a), the zone of constant strain is slightly longer for the specimens with the concrete surface prepared by grinding. For these specimens, shear failure within the concrete substrate was observed to propagate further along the length of the sheet prior to ultimate failure of the specimen. The hydro-blasted specimens without internal steel crossing the crack demonstrated the most localized failure, as shown in Figures 5-28 (b) and 5-29 (b), while the presence of steel appeared to allow a greater distribution of strain in the CFRP sheet prior to failure, as shown in Figures 5-28 (c) and 5-29 (c).
Unlike the strain distributions for the other bond specimens, the strain distribution for specimen R-45-H, shown in Figure 29 (b), does not include a zone of constant strain close to the crack. It is possible that for this specimen, the actual maximum strains were not recorded with the available instrumentation due to the excellent bond and highly localized load transfer between the CFRP sheets and the concrete.

It is widely recognized that the bond between the CFRP sheets and the concrete is excellent, and that the length of sheet over which load is effectively transferred, the effective bond length, $L_e$, is quite short. (Chajes et al. 1996, Maeda et al. 1997, Sato et al. 1997c) It should be noted however, that the maximum level of strain reached in the CFRP sheets prior to failure is typically only about 50% of the rupture strain of the material. Since the full strength of the CFRP sheets is not developed, the term “development length” is not used, but rather the term “effective bond length” is used to describe the zone of effective load transfer. Based on the strain distributions shown in Figures 28 and 29, the effective bond lengths observed for the Rectangular Bond Specimens were in the range of 75 to 100 mm.

Maeda et al. (1997) have introduced a relationship for predicting the effective bond length, $L_e$, as a function of the stiffness of the CFRP sheets, $E_r t_r$, which was presented in Chapter 2 as equation (2-9) and is as follows:

$$ L_e = \exp[6.134 - 0.580 \ln(E_r t_r)] $$

(5-2)
Where, 

\[ E_r = \text{modulus of elasticity of FRP sheet in GPa} \]

\[ t_r = \text{thickness of FRP sheet in mm} \]

Using Equation (5-2), an effective bond length of 70 mm is predicted for the single layer CFRP sheets used for the bond specimens in this experimental program. The relationship shown in Equation (5-2) therefore predicts an effective bond length on the lower end of the range observed in this experimental program.

As discussed in Chapter 2, the model for predicting bond capacity introduced by Maeda et al. (1997), is based on an extensive series of bond tests and includes a constant strain gradient, \( \frac{de}{dx} \), of 0.000110 mm\(^{-1}\) occurring over the effective bond length. Also based on bond test results, Sato et al. (1997c) have recorded constant strain gradients of about 0.000168 mm\(^{-1}\). Based on the strain distributions shown in Figures 28 and 29, the strain gradients for the Rectangular Bond Specimens tested in this experimental program are typically in the range of 0.000090 to 0.000105 mm\(^{-1}\). Steeper strain gradients do occur in some instances, such as the strain gradient of 0.000130 mm\(^{-1}\) observed for specimen R-45-H-S and shown in Figure 29 (c).

5.5.2 Strain Distribution Along Principal Fibres: Test Beams

Due to the excellent bond characteristics and very localized nature of load transfer between the concrete and the CFRP sheets, closely spaced 5 mm strain gauges were also
applied along the principal fibres of the CFRP sheets on the test beams. Figures 5-30 to 5-34 show the strain distribution along the principal fibres for representative CFRP sheets on test beams S-Diag-1, S-Diag-2, B-Diag100, B-Vert100 and B-Vert20, respectively. The strain distributions are plotted at various levels of applied shear load to illustrate the change in the distribution with increasing load levels.

For most of the CFRP strain distributions in Figures 5-30 to 5-34, the CFRP sheet crosses several shear cracks, as indicated by peaks in the strain at these crack locations. The localized nature of the load transfer and the short effective bond lengths are apparent in the strain distributions, particularly at lower levels of applied shear load. As the load increases, the strain in the CFRP sheet increases between the cracks.

A comparison of Figures 5-30 and 5-31 suggests that the strains measured in the double layer of diagonal sheets on beam S-Diag-2 are typically lower than those measured in the single layer of diagonal sheets on beam S-Diag-1. Similarly, the strain values for the thicker Type C sheets applied to beam B-Diag100 are lower than the strain values measured in the Type B sheets applied to beam S-Diag-1, as illustrated by a comparison of Figures 5-32 and 5-30. In general, an increase in the stiffness of the CFRP sheets results in a decrease in the measured strains.

For all test beams, with the exception of beam B-Diag100, a reduction in the CFRP strain is observed near the interior angle of the cross section just prior to the initiation of failure. This reduction in strain indicates that straightening of the CFRP sheets has begun and
marks the initiation of overall beam failure, which will be discussed in more detail in Chapter 6.

5.5.3 Strain Distribution Along Shear Cracks: Test Beams

Figure 5-35 illustrates the typical strain gauge layout for a test beam in Series S. The closely spaced 5 mm gauges were used to measure the distribution of strain along the principal fibres of the CFRP sheets, as discussed in the previous section. In the following section, the strain in the principal fibres of the sheets is considered once again, however the distribution of strain is examined along the shear crack, as shown in Figure 5-35, rather than along the length of the principal fibres. The effect of the type and configuration of CFRP sheets on the strain distribution along the shear crack is discussed in the following section. In Chapter 6, the measured strain distribution along the shear crack is used to determine the shear resisting force provided by the CFRP sheets during testing, and general trends in the measured strain distribution are used to develop a model for predicting the shear resistance provided by the CFRP sheets.

Shear failure occurred in a similar location for all beams, with the exception of beam S-Diag-CL where shear failure occurred in the end-block outside of the strengthened zone. For all beams, several inclined shear cracks occurred within the shear span in the zone of constant applied shear load. For the beams with CFRP sheets, the cracks were visible only in the gaps between sheets. It is possible to identify crack locations based on closely spaced strain measurements in the CFRP sheets, however, it is not possible to
identify which is the fatal shear crack causing failure of the beam. Therefore, for each beam, the strain distribution is examined at three different crack locations within the typical failure zone. The crack locations are labeled A, B, and C, as shown in Figure 5-35, and are similar for each beam. As observed during testing, the angle of the inclined cracks in the zone of shear failure is typically about 30° for all of the test beams.

**Strains in Single and Double Layer Diagonal Sheets** -- Figures 5-36 (a), (b) and (c) show the strain distribution along cracks A, B, and C, respectively, for the single layer of diagonal sheets on beam S-Diag-1. The reduced strain values observed at an applied shear load, $V_{\text{app}}$, of 230 kN are due to straightening of the CFRP sheets at the bottom of the web. As shown in Figures 5-37 (a), (b), and (c), a similar decrease in strain due to straightening of the CFRP sheets is observed for the double layer of diagonal sheets on beam S-Diag-2. The increased stiffness of the double layer sheets resulted in strain values that are only one-half to two-thirds the strain values obtained for the single layer of sheets.

**Effect of Horizontal CFRP Sheet** -- Figures 5-38 (a), (b) and (c) show the measured strain distributions along cracks A, B and C, respectively, for the diagonal sheets on beam S-Diag-H. Beam S-Diag-H includes a single layer horizontal sheet applied on top of a single layer of diagonal sheets. In general, the measured strain values are only slightly lower than those measured for beam S-Diag-1. As observed during testing, the straightening of the CFRP sheets was less extensive prior to failure, therefore the decrease in the measured strains is minimal. Due to difficulties in recording some of the
strain values just prior to ultimate failure, it is not possible to quantify precisely the extent to which the horizontal sheet controlled the straightening of the diagonal sheets.

**Effect of Clamping Scheme** -- The clamping scheme applied to beam S-Diag-CL effectively controlled the straightening of the diagonal CFRP sheets. As shown in Figures 5-39 (a) and (b), the measured strains in the clamped diagonal sheets reached higher levels than those recorded for the other beams. Over the entire depth of the cross section, the measured strains continue to increase with increasing levels of applied shear load until failure occurs outside of the strengthened zone. Since failure occurred outside of the strengthened zone, the full potential of this strengthening scheme was not realized and the strain in the CFRP sheets may not have reached the maximum values possible with the clamping scheme in place.

### 5.5.4 Prediction of Maximum Strain in CFRP Sheets

The maximum strain developed in CFRP sheets bonded to concrete can be predicted using the model introduced by Maeda et al. (1997) where a constant strain gradient, \( \frac{d\varepsilon}{dx} \), of 0.000110 m\(^{-1}\) is multiplied by the effective bond length, \( L_e \), as follows:

\[
\varepsilon_{f,\text{max}} = 0.000110 \times (L_e) \quad \text{(5-3)}
\]
The effective bond length is determined using Equation (5-2). Figure 5-40 shows the relationship between the maximum strain in the CFRP sheets, calculated using Equations (5-2) and (5-3), and the stiffness of the CFRP sheets. A comparison of the relationship proposed by Maeda et al. (1997), and various bond test results reported in the literature is also shown in Figure 5-40. Table 5-5 provides a summary of the bond test results collected from the literature, and the corresponding experimental parameters. Since the results of the bond tests conducted by Maeda et al. form the basis of the proposed model and were compared with the model in Figure 2-14, these particular results are not included again in Figure 5-40.

In order to simplify the equations introduced by Maeda et al., the following relationship between maximum CFRP strain and CFRP sheet stiffness is proposed:

\[
\varepsilon_{r_{\text{max}}} = \frac{1}{\sqrt{E_{r} t_{r} n_{r}}} \tag{5-4}
\]

Where \( E_{r} \) = modulus of elasticity of CFRP, in MPa

\( t_{r} \) = thickness of CFRP per layer, in mm

\( n_{r} \) = number of layers of CFRP sheets

It should be noted that the units used for the modulus of elasticity have been changed from GPa to MPa, as these units are used more commonly in North America. The relationship proposed in Equation (5-4) is shown in Figure 5-40, and compares well with
the relationship introduced by Maeda et al. and with the bond test data from the literature. In Figure 5-41, both the relationship introduced by Maeda et al. and the relationship proposed in Equation (5-4) are compared with the bond test results and beam test results obtained in this experimental program. The average maximum strains measured in the Rectangular Bond Specimens are all slightly above the more conservative prediction based on Equation (5-4) and slightly below the model introduced by Maeda et al.

For the beam tests, due to the shape of the cross section, tension in the CFRP sheets results in both peeling and shear stresses in the concrete substrate. The maximum strain in the CFRP sheets is therefore lower for the beam specimens, as shown in Figure 5-41. To account for the more complex state of stress and the existence of both shear and peeling stresses, the following modification of Equation (5-4) is proposed to predict the maximum strain in diagonal CFRP sheets on I-shaped girders:

$$\varepsilon_{f_{max}} = \frac{0.4}{\sin \alpha_f \sqrt{E_f t_f n_f}}$$

(5-5)

The term “sin $\alpha_f$”, reflects the fact that only the vertical component of the tensile force in the sheets will subject the concrete surface to peeling forces. As shown in Figure 5-41, the relationship proposed in Equation (5-5) compares well with the beam test results.

For beam S-Diag-CL, peeling and straightening of the diagonal CFRP sheets was prevented by the clamping scheme, and the maximum strains recorded in the CFRP
sheets were in the same range as those recorded for the Rectangular Bond Specimens, 0.006 to 0.008. Equation (5-4) therefore provides a conservative lower bound for the prediction of maximum strain in clamped diagonal CFRP sheets where peeling and straightening of the sheets is controlled.

It should be noted that the relationship introduced in Equation (5-4) can be used to predict the effective bond length by dividing the maximum CFRP strain by the constant strain gradient, $\frac{d\varepsilon}{dx}$, as follows:

$$L_e = \frac{\varepsilon_{r,\text{max}}}{(d\varepsilon/dx)}$$

(5-6)

If a constant strain gradient, $d\varepsilon/dx$, of 0.00009 mm$^{-1}$, as observed in this experimental program and reported in Section 5.5.1 is used, then substituting Equation (5-4) into Equation (5-6) results in the following relationship:

$$L_e = \frac{11,100}{\sqrt{E_r t_r n_r}}$$

(5-7)

Where $E_r$ = modulus of elasticity of CFRP, in MPa

t$_r$ = thickness of CFRP per layer, in mm

$n_r$ = number of layers of CFRP sheets
5.6 LOAD SHARING BETWEEN CFRP SHEETS AND STEEL STIRRUPS

In order to examine the load sharing relationship between the CFRP sheets and the internal steel stirrups, in both the bond specimens and the beam specimens, the strain was measured in both the sheets and the stirrups in approximately the same location. The expectation was that both load-resisting components crossing the same crack would undergo similar strain levels, and therefore the total applied tensile load could be proportioned to each component based on its stiffness. Based on test results, however, the assumption of equivalent strains does not hold for the entire range of loading, as will be demonstrated for both the bond specimens and the test beams.

At lower levels of applied load, strains measured in the CFRP sheets are typically higher than those measured in the steel stirrups at the same crack. A rational explanation for this behaviour is provided in Section 5.6.3, by considering compatibility of deformations at the crack and the different bond properties and load transfer mechanisms observed for CFRP sheets and steel reinforcing bars.

5.6.1 Load Sharing in Bond Specimens

For the bond specimens with internal steel crossing the crack, ratios of CFRP sheet strain to steel strain at increasing levels of applied load are shown in Figure 5-42. As expected, the strain ratio is approximately equal to 1.0 prior to cracking. The CFRP sheets and the internal steel are fully bonded to the concrete at this stage, and each of the three materials
undergoes equivalent strain. Load sharing is therefore based on the ratio of the stiffness of each material at this stage.

After cracking, however, the strain ratio is greater than 1.0 for all three of the bond specimens. The ratio is highest for the Rectangular Bond Specimens, with values ranging from 3.0 to 3.7. For the Single-Flanged Bond Specimen, the ratio of CFRP sheet strain to steel strain is approximately 1.5.

The higher strain observed in the FRP sheets when compared to the steel bars, as reflected by strain ratios greater than 1.0, is attributed to the improved bond and more localized force transfer in the CFRP sheets when compared to the steel. This behaviour is discussed in more detail in Section 5.6.3.

5.6.2 Load Sharing in Test Beams

In order to examine the load sharing relationship between the CFRP sheets and steel stirrups, strains in each component were monitored in approximately the same location, as illustrated in Figures 5-43 and 5-45. In this section, the relationship between the vertical strain component in the CFRP sheets and the strain in the steel stirrups is examined at the bottom of the web and at the top of the web for each beam.

For each individual beam, the strains measured in the CFRP sheets at several strain gauge locations along the bottom of the web are found to be similar, as shown in
Figures 5-43 (a) through 5-43 (e). Therefore, average strains are used to represent the strain in the CFRP sheets at the bottom of the web. Similarly, average stirrup strains are found to be representative of individual stirrup strains measured at the bottom of the web for each beam, as shown in Figures 5-43 (a) through 5-43 (e).

For the beams with diagonal CFRP sheets only, the ratios of average sheet strain to average stirrup strain at the bottom of the web are shown in Figure 5-44 (a). As the level of applied shear load increases, a gradual decrease in the ratio of sheet strain to stirrup strain is observed for each beam, with the exception of beam B-Diag-100. At an applied shear load of 190 kN, straightening of the CFRP sheets is initiated for beams S-Diag-1 and S-Diag-2, resulting in reduced strain in the CFRP sheets and a more rapid decrease in the strain ratio with increasing levels of applied shear load. By comparison, the ratio of sheet strain to stirrup strain for the beam with clamped diagonal sheets, S-Diag-CL, is constant until failure, as shown in Figure 5-44 (b). For beam S-Diag-H, with a horizontal sheet on top of the diagonal sheets, the strain ratio is constant until the more controlled and less extensive straightening of the diagonal sheets results in a gradual decrease in the strain ratio.

The strains measured in the CFRP sheets at several strain gauge locations along the top of the web are found to be similar, for each individual beam, as shown in Figures 5-45 (a) through 5-45 (b). Average strains are therefore adequate to represent the strain in the CFRP sheets at the top of the web for these beams. Similarly, average stirrup strains are representative of individual stirrup strains measured at the top of the web for each beam,
as shown in Figures 5-45 (a) through 5-45 (d).

The ratios of sheet strain to stirrup strain at the top of the web, for all of the beams where data is available, are shown in Figure 5-46. For all of the beams, the strain ratio at the top of the web is approximately constant until failure.

Since the CFRP sheets and the steel stirrups provide shear-resisting forces all along the inclined shear cracks, at both the bottom and the top of the web, the average behaviour is considered. For beams where strain ratios are available at both the top and the bottom of the web, the average of the top and bottom strain ratios are shown in Figure 5-47. Based on the strain ratios plotted in Figures 5-44 and 5-47, a value of 1.5 is proposed for the strain ratio occurring prior to straightening of the CFRP sheets.

5.6.3 Effect of Bond Properties on Strain Measurements at a Crack

The relationship between the strain in the CFRP sheets and the strain in the steel stirrups is important for the design of strengthening schemes where each component is required to carry a portion of the applied loading. If failure of the strengthened member is controlled by the initiation of failure in one component, the CFRP sheets for example, then in some cases it may be unconservative to assume that the steel strain has reached levels as high as the CFRP sheets at the initiation of failure.
In addition to the results obtained for both the bond specimens and beams tested in this experimental program, several other researchers have recorded higher strain values in the CFRP sheets when compared to the strains measured in the steel stirrups. (Miyauchi et al. 1997, Sato et al. 1996, Uji 1992) This behaviour has been attributed to the superior bond properties of the FRP sheets when compared to the steel bars. (Uji 1992, Sato et al. 1996)

In the following section, a rational explanation for the observed behaviour is proposed, based on the observation that load transfer between the extremely thin and wide FRP sheets and the concrete must be significantly different than the load transfer between a discrete bar and the concrete surrounding the bar circumference.

The idealized load transfer mechanism or bond behaviour between a steel reinforcing bar and concrete in a cracked reinforced concrete member subjected to tensile force is shown in Figure 5-48. On either side of the crack, a combination of slip and load transfer occurs. As load is transferred into the surrounding concrete, the strain in the concrete, $\varepsilon_c$, increases between cracks while the strain in the steel bar decreases from the maximum level at the crack, $\varepsilon_{s\text{ max}}$. The crack width, $w$, can be determined as the difference between the strain in the steel and the concrete over the distance between cracks, $L$, as follows: (MacGregor 1997)

$$w = \int_0^L (\varepsilon_s - \varepsilon_c) \, dx$$  \hspace{1cm} (5-8 a)

The bond mechanism between the FRP sheets and the concrete on either side of a crack has been discussed in detail in Section 5.5. Similar to Equation (5-8 a), the crack width
for a concrete member reinforced with FRP sheets and subjected to tension can be determined as follows:

\[ w = \int_{0}^{L} (\varepsilon_r - \varepsilon_c) \, dx \]  \hspace{1cm} (5-8b)

Figure 5-49 (a) illustrates a concrete member subjected to tension and reinforced with both FRP sheets and steel bars. Due to the excellent bond characteristics of the FRP sheets and the short effective bond length, \( L_e \), when compared to the longer load transfer length of the steel bar, the maximum strain at the crack may be higher in the FRP sheet. If the FRP sheet and the steel bar are both bonded to the concrete at some point between the cracks, then for compatibility, the crack width as determined by Equations (5-8a) and (5-8b) must be equivalent. Therefore, although the maximum strain in the CFRP sheets may exceed the maximum strain in the steel bar, the average strain over the distance \( L \) must be equal, as shown below:

\[
\frac{\int_{0}^{L} (\varepsilon_s - \varepsilon_c) \, dx}{L} = \frac{\int_{0}^{L} (\varepsilon_r - \varepsilon_c) \, dx}{L} \]  \hspace{1cm} (5-9)

It is reasonable to assume that there is a limit to the discrepancy that can exist between the maximum FRP strain, \( \varepsilon_{r \text{ max}} \), and the maximum steel strain, \( \varepsilon_{s \text{ max}} \), since this discrepancy would create shear strain in the concrete.

If tensile force can be applied to the steel bar exclusively by mechanical anchorage at the
ends of the bar, similar to stirrups in a beam, then the steel bar may become completely debonded from the concrete as cracking progresses. In this case, the strain distribution in the FRP sheets and the steel bar could be similar to that shown in Figure 5-49 (b). For the concrete members shown in both Figures 5-49 (a) and (b), compatibility requirements are satisfied, only if the average strain in the FRP sheets and the steel bars is equivalent.

Since CFRP sheets are extremely thin and are typically bonded over a significant width of concrete substrate, the bond behaviour is expected to be markedly different than the bond behaviour of a discreet steel bar where load transfer occurs only around the circumference of the bar. The cross section shown in Figure 5-49 (a) does not adequately reflect the difference in the bond behaviour, since the comparatively large width, \( w_b \), of the FRP sheets is not shown. Figure 5-49 (c) shows part of a larger idealized concrete member in three dimensions, where the discreet steel bars are distributed over the dimension \( Z \) of the member at a typical spacing of 200 mm.

For the larger concrete member shown in Figure 5-49 (c), compatibility does not require that the crack width be equivalent all along the length, \( Z \), of the member. It is reasonable to assume that the crack width along the length \( Z \) is mainly controlled by the widely distributed FRP sheets, except at discreet locations where a steel bar acts with the FRP sheets to restrain the opening of the crack. At the discreet bar locations, the crack width may be reduced slightly, as well as the average strain in both the FRP sheets and the steel bar. For the overall member, however, the average crack width is larger, and the average strain in the FRP sheets is larger as well.
### Table 5-1 Parameters and Test Results: Series B

<table>
<thead>
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<th>Layer 1 Config.</th>
<th>Layer 2 Config.</th>
<th>gap (mm)</th>
<th>s_f (mm)</th>
<th>CFRP Type</th>
<th>f'_c (MPa)</th>
<th>V_{test} (kN)</th>
<th>V_{test} / V_{control}</th>
<th>Beam Mark</th>
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### Table 5-2 Parameters and Test Results: Series S

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### Table 5-3 Parameters and Test Results: Rectangular Bond Specimens

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<th>Surface Prep.</th>
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<th>Crack Angle</th>
<th>Ultimate Load (kN)</th>
<th>Measured Ave. Strain</th>
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### Table 5-4 Parameters and Test Results: Single-Flanged Bond Specimens

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* Shortest Distance from Interior Angle to Crack
### Table 5-5 (a) Summary of Bond Test Results from the Literature: Part a

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*D = Drimoussis and Cheng 1994*  
*A = Alexander and Cheng 1996*  
*S = Sato et al. 1997c*
Table 5-5 (b)  Summary of Bond Test Results from the Literature: Part b

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I = Iketani and Jinno 1997
Y = Yoshizawa et al. 1996
C = Chajes et al. 1996
Figure 5-1 (a) Beam B-Control at Failure

Figure 5-1 (b) Beam B-Control: Stirrup Straightening
Figure 5-2  Beam B-Control Just Prior to Failure

Figure 5-3  Beam B-CL at Failure

Figure 5-4  Beam B-CL Just Prior to Failure
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Figure 5-6  Beam B-Vert20 at Failure
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Figure 5-7(b) Beam B-Vert100 After Removal of CFRP Sheets
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Figure 5-9 Beam B-Diag100 at Failure
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Figure 5-11  Stirrup Strain vs Applied Shear Load: Series B Beams with Diagonal Sheets
Figure 5-12 (a) Beam B-Vert-H at Failure

Figure 5-12 (a) Beam B-Vert-H with FRP Sheets Removed
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Figure 5-13 (b) Beam B-Diag-H with FRP Sheets Removed
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Figure 5-15  Beam S-NoFRP at Failure
Figure 5-16  Beam S-Control: Premature Failure of CFRP Strips

Figure 5-17  Beam S-Control at Failure
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Figure 5-19  CFRP Strip at Failure: Series S
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Figure 5-20 (b) Beam S-Diag-2 at Failure
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Figure 5-21 (b) Beam S-Diag-H Right of First Loading Point at Failure
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Figure 5-23 (b) Stirrup Strain at Bottom of Web vs Applied Shear Load: Series S
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Figure 5-25 (b) Failure of Rectangular Bond Specimen R-45-G
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Figure 5-26 (b) Failure of Rectangular Bond Specimen R-45-H-S
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Figure 5-28 (c)  
Strain Distribution Along Principal Fibres:  
Rectangular Bond Specimen R-0-H-S
Figure 5-29 (a)  Strain Distribution Along Principal Fibres: Rectangular Bond Specimen R-45-G
Figure 5-29 (b) Strain Distribution Along Principal Fibres: Rectangular Bond Specimen R-45-H
Figure 5-29 (c)  Strain Distribution Along Principal Fibres: Rectangular Bond Specimen R-45-H-S
Figure 5-30 Strain Distribution Along Principal Fibres: Beam S-Diag-1

Figure 5-31 Strain Distribution Along Principal Fibres: Beam S-Diag-2
Figure 5-32 Strain Distribution Along Principal Fibres: Beam B-Diag100
Figure 5-33 Strain Distribution Along Principal Fibres: Beam B-Vert100

Figure 5-34 Strain Distribution Along Principal Fibres: Beam B-Vert20
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Along Shear Cracks A, B and C
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Fig. 5-36 (b) CFRP Strain Distribution: Crack B, Beam S-Diag-1

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Fig. 5-37 (b) CFRP Strain Distribution: Crack B, Beam S-Diag-2

Fig. 5-37 (c) CFRP Strain Distribution: Crack C, Beam S-Diag-2
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Fig. 5-38 (b) CFRP Strain Distribution: Crack B, Beam S-Diag-H

Fig. 5-38 (c) CFRP Strain Distribution: Crack C, Beam S-Diag-H
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Figure 5-42 (b) Ratio of Force in FRP Sheets to Force in Steel Bar: Single-Flanged Bond Specimen SF-0-50-S
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Figure 5-43 (b)  Strain Measured in FRP Sheets and Steel Stirrups: at Bottom of Web on Beam S-Diag-2
Figure 5-43 (c) Strain Measured in FRP Sheets and Steel Stirrups: Bottom of Web on Beam B-Diag100

Figure 5-43 (d) Strain Measured in FRP Sheets and Steel Stirrups: at Bottom of Web on Beam S-Diag-H
Figure 5-43 (e)  
Strain Measured in FRP Sheets and Steel Stirrups:  
Bottom of Web on Beam S-Diag-CL
Figure 5-44 (a) Ratio of CFRP Sheet Strain to Stirrup Strain at Bottom of Web

Figure 5-44 (b) Ratio of CFRP Sheet Strain to Stirrup Strain at Bottom of Web
Figure 5-45 (a) Strain Measured in FRP Sheets and Steel Stirrups: Top of Web on Beam S-Diag-1

Figure 5-45 (b) Strain Measured in FRP Sheets and Steel Stirrups: at Top of Web on Beam S-Diag-2
Figure 5-45 (c) Strain Measured in FRP Sheets and Steel Stirrups: Top of Web on Beam B-Diag100

Figure 5-45 (d) Strain Measured in FRP Sheets and Steel Stirrups: at Top of Web on Beam S-Diag-H
Figure 5-46  Ratio of CFRP Sheet Strain to Stirrup Strain at Top of Web

Figure 5-47  Ratio of CFRP Sheet Strain to Stirrup Strain: Average of Top and Bottom Ratios
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Figure 5-49 (a)  Idealized Bond Behaviour: FRP Sheets and Steel Bar Reinforcement
Figure 5-49 (b)  Idealized Bond Behaviour: FRP Sheets and Debonded Steel Bar

Figure 5-49 (c)  Idealized Bond Behaviour for Part of a Concrete Member Reinforced with FRP Sheets and Steel Bars
RATIONAL MODEL FOR SHEAR CAPACITY OF I-SHAPED CONCRETE GIRDERS STRENGTHENED WITH FRP SHEETS

6.1 GENERAL

The shear strength, $V_n$, of a reinforced concrete beam with externally bonded FRP sheets can be calculated as the sum of the shear resisting contributions of the concrete, $V_c$, the steel stirrups, $V_{se}$, and the FRP sheets, $V_{f_{\text{max}}}$. As observed during testing, for I-sections, failure is initiated by straightening of the CFRP sheets due to an outward force subjecting the concrete substrate to both peeling and shear stresses. After straightening of the FRP sheets is initiated by failure in the concrete substrate, a decrease in the strain in the FRP sheets is observed, and a corresponding decrease in the shear resistance of the FRP sheets occurs. The nominal shear strength provided by the FRP sheets, $V_{f_{\text{max}}}$, is based on the maximum FRP contribution which occurs just prior to straightening of the FRP sheets.

In this chapter, the contribution to shear resistance provided by the FRP sheets and the steel stirrups, at increasing levels of applied shear load, is determined experimentally.
based on strain measurements in each component. Analysis of the shear resistance provided by each component serves to clarify the behaviour observed during testing, and facilitates the development of the rational model for predicting the shear capacity of the strengthened beam.

In the model, the shear resistance provided by the FRP sheets is calculated using an expression to predict the maximum strain developed in the FRP sheets, and an FRP strain distribution model introduced for I-shaped sections. The stirrup contribution to shear resistance is calculated using the basic truss model, however the strain in the stirrups is determined based on the strain in the FRP sheets, and may be less than the yield strain. Since the truss model is used to predict both the FRP sheet and the steel stirrup contributions, the angle used for the inclined concrete compressive struts significantly affects the predicted values.

In the model, an inclination angle of 30 degrees is used to determine the FRP sheet and steel stirrup contributions and the concrete contribution is predicted using the ACI method. In this chapter, it is also shown that the use of the ACI 45 degree truss model is overly conservative and severely underestimates the contribution of both the FRP sheets and the steel stirrups. The modified sectional-truss method introduced by Ramirez and Breen (1991) includes a concrete contribution in combination with a variable angle truss model and is also used for comparison. The lower limit of 25 degrees for the compressive strut inclination angle is used when applying the modified sectional-truss method. In order to assess the reliability of the proposed rational model, the shear
resistance of the Series B and Series S test beams is predicted using the model and compared with actual test results.

6.2 SHEAR RESISTING COMPONENTS DETERMINED EXPERIMENTALLY

In this section, the strains measured in the CFRP sheets and steel stirrups crossing the shear cracks are used to determine the shear resisting force provided by each component during testing. For each of the Series S beams, the shear resistance provided by each component is determined at increasing levels of applied shear load until failure and is discussed in the following sections.

6.2.1 Procedure for Determining Shear Resistance Experimentally

As described in Section 5.5.3, the strain in the CFRP sheets is recorded along three different shear cracks within the typical failure zone. In this section, the strain measured in both the CFRP sheets and the steel stirrups crossing the shear cracks is used to determine the shear resistance provided by each component. Figure 6-1 shows the location of strain measurements in both the sheets and the stirrups, as well as the three crack locations labeled A, B, and C. Since the strain gauges are evenly spaced along each shear crack, the average of the strains measured along the crack is used to calculate the contribution to shear resistance of each component. Where the crack passes midway between two strain gauges on a given stirrup, the average measured strain is used to
represent a strain measurement at that particular location along the crack.

The cross sectional area of the steel stirrups and the CFRP sheets, the stress-strain relationships of the materials, and the average strain along the shear crack are used to calculate the shear resisting force for each component as follows:

\[
V_s = \varepsilon_{s\text{ave}} E_s A_v \frac{d (\cot \theta)}{s}\tag{6-1}
\]

\[
V_f = \varepsilon_{f\text{ave}} E_f 2n_f t_f w_f d_f (\cot \theta + \cot \alpha_f) \sin \alpha_f \tag{6-2}
\]

Where,

- \(V_s\) = shear resistance provided by steel stirrups
- \(V_f\) = shear resistance provided by FRP sheets
- \(\varepsilon_{s\text{ave}}\) = average measured strain in stirrups
- \(\varepsilon_{f\text{ave}}\) = average measured strain in FRP sheets
- \(E_s\) = modulus of elasticity of steel stirrups
- \(E_f\) = modulus of elasticity of FRP sheets
- \(A_v\) = area of stirrup legs per stirrup
- \(n_f\) = number of layers of FRP sheets per one side of beam
- \(t_f\) = thickness of one layer of FRP sheets
- \(w_f\) = width of one FRP sheet perpendicular to principal fibres
- \(s\) = spacing of stirrups along longitudinal beam axis
- \(s_f\) = spacing of FRP sheets along longitudinal beam axis
- \(d\) = effective depth of cross section from extreme compression fibre to centroid of tension reinforcement
The shear resistance provided by the CFRP sheets, as determined using Equation (6-2), is illustrated schematically in Figure 6-2.

For each beam, the shear resistance provided by the FRP sheets, $V_f$, was calculated at the three crack locations, A, B, and C. Because for each beam, the values of $V_f$ calculated at each crack, $(V_f)_A$, $(V_f)_B$ and $(V_f)_C$, were typically found to be in good agreement with each other, the average value of $V_f$ is determined for each beam as follows:

$$V_f = [(V_f)_A + (V_f)_B + (V_f)_C] / 3 \quad (6-3)$$

The average values for $V_f$ are calculated for each beam and are presented and discussed in the following sections.

Similarly, the shear resistance provided by the stirrups, $V_s$, was calculated at each crack and the values of $(V_s)_A$, $(V_s)_B$ and $(V_s)_C$, were found to be in good agreement for each beam. The average values for $V_s$ are calculated for each beam and are presented and discussed in the following sections.
In accordance with the traditional ACI approach to shear design (ACI 318, 1995) the shear resisting force in the FRP sheets, $V_f$, and the shear resisting force in the stirrups, $V_s$, each resist a portion of the applied shear force, $V_{app}$, while the remainder of the applied shear force is resisted by the concrete contribution, $V_c$. The concrete contribution to shear resistance can therefore be determined by subtracting the total contribution provided by the shear reinforcement from the applied shear force as follows:

$$V_c = V_{app} - (V_s + V_f) \quad (6-4)$$

Where $V_f$ and $V_s$ are based the average shear resistance provided by the FRP sheets and the steel stirrups crossing three shear cracks.

### 6.2.2 Experimental Shear Resisting Components for Beam S-Diag-1

The shear resisting forces provided by the steel stirrups, $V_s$, and the single layer of diagonal CFRP sheets, $V_f$, for beam S-Diag-1 are plotted versus applied shear load and shown in Figure 6-3. The concrete contribution, $V_c$, is calculated using Equation (6-4), and is also shown in Figure 6-3.

As illustrated in Figure 6-3, the maximum value of the FRP contribution, $V_{f_{max}}$, occurs at an applied shear load of 190 kN and represents the initiation of straightening of the FRP sheets. The maximum strain in the FRP sheets at the initiation of straightening is approximately 0.004, as shown previously in Figure 5-36. The FRP contribution begins
to drop off as straightening continues. At this stage, the stirrup contribution increases rapidly, until complete failure of the beam occurs at an applied shear load of 234 kN.

6.2.3 Experimental Shear Resisting Components for Beam S-Diag-2

For beam S-Diag-2 with a double layer of diagonal CFRP sheets, the shear resisting forces provided by the steel stirrups, $V_s$, and the FRP sheets, $V_f$, are shown in Figure 6-4, along with the concrete contribution, $V_c$. Similar to beam S-Diag-1, the FRP contribution for beam S-Diag-2 also reaches a maximum at the initiation of sheet straightening, followed by a decrease in the FRP contribution. The maximum FRP contribution, $V_{f,\text{max}}$, occurs at an applied shear load of 195 kN. The stirrup contribution, $V_s$, also increases rapidly at this stage, as was observed for beam S-Diag-1.

6.2.4 Experimental Shear Resisting Components for Beam S-Diag-H

Figure 6-5 shows the shear resisting forces provided by the steel stirrups and the diagonal FRP sheets, as well as the concrete contribution, for the beam with a horizontal sheet applied on top of diagonal CFRP sheets, S-Diag-H. As illustrated in Figure 6-5, the FRP contribution, $V_f$, approaches a constant value at an applied shear load of about 203 kN. Correspondingly, an increase in the slope of the $V_s$ versus applied shear load curve is observed at this load level. Since several strain measurements in the FRP sheets were unavailable beyond an applied shear load level of 221 kN, the remainder of the CFRP contribution up to ultimate beam failure could not be documented.
Straightening of the FRP sheets at the bottom of the web was observed for beam S-Diag-H, but appeared to be more controlled than the straightening behaviour observed for the beams without the horizontal sheet, S-Diag-1 and S-Diag-2. Correspondingly, $V_{f_{\text{max}}}$ remained relatively constant for beam S-Diag-H, rather than dropping off significantly. Since the straightening behaviour was controlled to some extent, the stirrups reached yield and $V_s$ reached a maximum value of $V_{s y}$ without a significant drop in the FRP contribution, $V_{f_{\text{max}}}$. However, as described above, the exact value for $V_f$ is not available at applied shear load levels greater than 221 kN.

6.2.5 Experimental Shear Resisting Components for Beam S-Diag-CL

For beam S-Diag-CL with a single layer of clamped diagonal CFRP sheets, the shear resisting forces provided by the steel stirrups, $V_s$, and the FRP sheets, $V_f$, are presented in Figure 6-6 along with the concrete contribution, $V_c$, which is calculated based on these average contributions.

As shown in Figure 6-6, the $V_f$ versus applied shear load curve for beam S-Diag-CL continues to increase throughout the loading of the beam until failure occurs outside of the strengthened zone. No decrease in $V_f$ or change in slope of $V_f$ or $V_s$ is observed. Because failure occurred outside of strengthened zone and the zone of strain measurement, maximum values for $V_f$ and $V_s$ likely do not represent the maximum shear capacity of these components.
6.2.6 Comparison of Shear Resisting Components

**CFRP Sheets** -- A comparison of the shear resisting forces provided by the diagonal FRP sheets, $V_f$, for all Series S beams is presented in Figure 6-7. Although the cross sectional area of CFRP for beam S-Diag-2 is twice that for beam S-Diag-1, the shear resisting force is not significantly higher with the addition of a second layer of CFRP. The application of a horizontal sheet on top of a single layer of diagonal FRP sheets did not significantly increase the FRP contribution for beam S-Diag-H. However, the horizontal sheet does appear to have controlled the straightening of the FRP sheets to some extent and prevented a significant drop in $V_f$ for this beam. The maximum value of $V_f$ is greatest for the beam with clamped diagonal sheets, S-Diag-CL, however, since failure occurred outside of the strengthened zone, this value may not represent the maximum shear resistance of clamped diagonal sheets.

**Steel Stirrups** -- Figure 6-8 shows a comparison of the shear resistance provided by the steel stirrups for all of the Series S beams, including beam S-Control and the unstrengthened beam, S-NoFRP. At any level of applied shear load after cracking, a reduced stirrup contribution, $V_s$, is observed for the beams strengthened with FRP sheets. A change in the slope of the $V_s$ versus applied shear load curve is apparent for all of the beams with the exception of the beam with clamped diagonal sheets, S-Diag-CL. For the beams strengthened with FRP sheets, the $V_s$ versus applied shear load curves are generally in good agreement, with the values for $V_s$ slightly lower for beams S-Diag-2 and S-Diag-CL.
Concrete -- The concrete contributions, $V_c$, are plotted for the Series S beams with diagonal FRP sheets and are shown in Figure 6-9. Consistent with many models for shear resistance (ref), the concrete contribution is generally constant for all of the beams. Beam S-Diag-CL, with the clamping scheme applied to the top and bottom of the web, appears to have a slightly higher concrete contribution than the other three beams, as shown in Figure 6-9.

6.3 PREDICTING THE CONCRETE CONTRIBUTION TO SHEAR RESISTANCE

In practice, the most significant difference between all of the shear prediction models described in Chapter 3 is the method for determining the shear resistance provided by the concrete and the angle of the inclined shear cracks. In this section, these methods are discussed further and are evaluated based on a comparison with observations made during testing.

6.3.1 Shear Cracking

In the traditional ACI approach to shear design, the shear load at which inclined shear cracks first occur is considered equivalent to the shear resistance provided by the concrete. It is assumed that the concrete contribution remains constant from first cracking until ultimate. As discussed in Chapter 3, predictions for the shear load causing web shear cracking and the load causing flexural shear cracking are compared, and the
lowest value is selected as the concrete contribution to shear resistance. The expressions used to predict web shear cracking, $V_{cw}$, and flexural shear cracking, $V_{ci}$, are as follows (ACI-318-M95):

\[
V_{cw} = 0.3 \left( \sqrt{f_c'} + f_{pc} \right) b_w d + V_p \quad (6-5 \ a)
\]

\[
V_{ci} = 0.05 \sqrt{f_c'} b_w d + V_d + V_i M_{cr} / M_{max} \quad (6-5 \ b)
\]

For each of the test beams in this experimental program, Equations (6-5 a) and (6-5 b) are used to predict the shear cracking load at various cross sections along the length of the beam, as shown in Figures 6-10 to 6-16. Using significant increases in stirrup strain measurements as an indication of first cracking, the shear cracking load observed during testing is plotted at various stirrup locations within the shear span for each beam. In Figures 6-10 to 6-16, the observed experimental values are compared with the shear cracking loads predicted using the ACI approach and found to be in fairly good agreement.

As discussed in Chapter 3, the following equation is introduced by Ramirez and Breen (1991) to predict the shear cracking load and the concrete contribution for prestressed beams as part of the modified sectional-truss method:

\[
V_c = K (0.166) \sqrt{f_c'} b_w d \quad (6-6)
\]
Where, \( K = \sqrt{1 + \frac{f_{pc}}{f_t}} \) and \( 1.0 \leq K \leq 2.0 \)

\( f_t = 0.166 \sqrt{f_c'} \)

\( f_{pc} = \) compressive stress at neutral axis

A comparison of the shear cracking loads and concrete contributions predicted for the test beams in this experimental program, using Equations (6-5) and (6-6), was provided in Chapter 3, Figure 3-6. The two methods will be compared further later in this chapter.

### 6.3.2 Influence of Shear Reinforcement on Concrete Contribution

Since externally bonded CFRP sheets are extremely thin, it is considered that FRP sheets do not significantly affect the load level at which shear cracking begins. (Uji 1992) Tests conducted by Drimoussis and Cheng (1994) have confirmed this postulation. A comparison of Figures 6-10 through 6-16 suggests that the presence of CFRP sheets did not significantly affect the shear cracking load for the beams in this experimental program. It appears that the compressive strength of the concrete, \( f_c' \), as listed in Tables 3-2 and 3-3 and shown in Figures 6-10 to 6-16, had a greater effect on the shear cracking load than the presence of CFRP sheets.

Although the concrete contribution to shear resistance predicted using Equations (6-5) and (6-6) is clearly not influenced by the amount of shear reinforcement, it is recognized that the presence of shear reinforcement has a beneficial effect on the concrete contribution, \( V_c \). The strain in the shear reinforcement at ultimate is limited in the ACI
code, in order to control crack widths and ensure that the integrity of the concrete contribution to shear resistance is maintained. It is therefore recommended that the average strain in the CFRP sheets, \( \varepsilon_{\text{ave}} \), be limited to 0.004. (ACI 440-F 1998, Khalifa et al. 1998)

### 6.3.3 Angle of Inclined Shear Cracks

A major difference between the three shear prediction models described in Chapter 3, is the angle of the inclined shear cracks. While the traditional ACI approach uses the conservative 45 degree truss model, the variable angle truss model and the modified sectional-truss model introduced by Ramirez and Breen (1991) both allow for variation in the angle of the inclined struts, within limits.

Both the ACI approach and the modified truss method rely on an assessment of the state of stress in the web, and at the extreme tension fibre, to predict shear cracking in prestressed concrete beams. Similarly, by an analysis of the principal stresses at various locations on the web of the prestressed member, the angle of the principal compressive stresses just prior to cracking can be determined. Figures 6-17 (a) and (b) show the angle of the principal compressive stresses just prior to cracking, as calculated at three different locations on the web for each beam in Series B and Series S, respectively.

During testing, it was observed that the first inclined shear cracks developed by the propagation of existing flexural cracks into the web. Upon reaching the top of the tension...
flange, just below the base of the web, the vertical shear cracks became inclined and propagated into the web. The initial shear cracks typically propagated to somewhere between the base of the web and the mid-height of the web before stabilizing. Further crack propagation above mid-height of the web typically did not occur until close to ultimate.

The principal compressive stress angles shown in Figures 6-17 (a) and (b) were calculated for the zone of initial shear cracking between the base of the web and mid-height of the web. Based on Figures 6-17 (a) and (b), the average value for the principal compressive stress angles in this zone is about 30 degrees. Although the angles shown in Figures 6-17 (a) and (b) were predicted using elastic theory, the average value of 30 degrees is in good agreement with the observations made during testing.

Based on the discussion above, a shear crack angle of 30 degrees is used for the proposed rational model. As recommended by Ramirez and Breen (1991), a lower limit of 25 degrees for the compressive strut inclination angle in prestressed concrete beams is also considered when applying the modified sectional-truss method. And finally, it is also shown that the use of the 45 degree truss model is overly conservative and as expected, severely underestimates the contribution of both the FRP sheets and the steel stirrups.
6.4 SHEAR RESISTANCE OF CFRP SHEETS

As described in Chapter 3, most models for predicting the shear capacity of reinforced concrete beams are based on truss models, where the contribution of shear reinforcement is the total force provided by the shear reinforcement over the horizontal projection of one truss panel. For models where the angle of the diagonal compressive struts, \( \theta \), is assumed to be equal to the angle of shear crack, \( \theta_s \), the contribution of the shear reinforcement can also be described as the force in the shear reinforcement crossing the inclined shear crack, as shown in Figure 6-2. If the shear reinforcement is at an angle, \( \alpha \), less than 90 degrees, the vertical component of the force in the shear reinforcement must satisfy equilibrium in the vertical direction. A general expression for the shear resistance provided by externally bonded FRP sheets, \( V_{f_{\text{max}}} \), applied at any angle, \( \alpha_f \), was provided in Equation (6-2).

In determining the contribution to shear resistance provided by the FRP sheets, the values for \( d_f \) and \( e_{f_{\text{ave}}} \) depend on the mode of failure, and are determined based on the FRP sheet configuration and the shape of the cross section. The first mode of failure shown in Figure 6-18 (a), rupture of the FRP sheets, typically occurs due to stress concentrations at stress levels lower than the ultimate tensile strength of the FRP sheet. For applications where the FRP sheets cannot be completely wrapped around the beam cross section, the bond between the FRP sheet and the concrete is critical. Failure of the bond mechanism is the second mode of failure shown in Figure 6-18 (b) and typically occurs due to shear-
tension failure within the concrete substrate. For the I-shaped section shown in Figure 6-18 (c), the tensile forces developed in the FRP sheets subject the concrete substrate to peeling forces as well as shear forces. Failure typically occurs within the concrete substrate due to straightening of the FRP sheets, prior to the development of a uniform strain distribution in the sheets.

For the first two modes of failure shown in Figure 6-18 (a) and (b), methods for determining $d_r$ and $\varepsilon_{r,ave}$ have been introduced by others and discussed in detail in Chapter 3 (Triantafillou 1998, Khalifa et al. 1998). For I-shaped cross sections, the average strain in the FRP sheets at failure, $\varepsilon_{r,ave}$, is based on the strain distribution model shown schematically in Figure 6-18 (c) and described in more detail in the following section.

6.4.1 FRP Strain Distribution Model for I-shaped Sections

A model for the strain distribution in FRP sheets bonded to I-shaped cross sections is introduced in this section. The model FRP strain distribution is based on the strain distributions measured during testing, and occurring at the initiation of failure due to straightening of the FRP sheets.

As shown in Figure 6-18 (c), a constant strain of $\varepsilon_{r,max}$ extends from the bottom of the effective depth to mid-height of the web, and decreases to zero at the top of the effective depth of the FRP sheet. Based on the geometry of the strain distribution, the average
FRP strain for I-shaped sections can be determined as follows:

$$\varepsilon_{f\text{ ave}} = \varepsilon_{f\text{ max}} \frac{(d/2) + 0.5 (d_r - d/2)}{d_r}$$

(6-7)

The shape of the strain distribution model was generally found to be consistent for all of the beams tested in this experimental program, however the maximum strain in the FRP sheets, $\varepsilon_{f\text{ max}}$, varied depending upon the stiffness and configuration of the FRP sheets. Methods for predicting the maximum strain in the FRP sheet, $\varepsilon_{f\text{ max}}$, as a function of the stiffness and configuration of the FRP sheets are described in Sections 6.4.2 and 6.4.3.

As observed during testing, the strain distribution shown in Figure 6-18 (c) occurs just prior to straightening of the FRP sheets. After straightening of the FRP sheets is initiated, the average strain in the FRP sheets is reduced and a decrease in the shear resistance of the FRP sheets occurs. Therefore, the nominal shear strength provided by the FRP sheets, $V_{r\text{ max}}$, is calculated using Equation (6-2) and Equation (6-7) and is based on the maximum FRP contribution which occurs just prior to straightening of the FRP sheets.

6.4.2 Effect of CFRP Stiffness on Maximum FRP Sheet Strain

As discussed in Chapters 2 and 3, the trend observed in the literature for both bond and beam tests, is that an increase in the FRP sheet stiffness results in an increase in the
tensile force in the sheets which is not directly proportional to the increase in the
stiffness. Put more simply, if bond failure is the controlling mechanism, the addition of a
second layer of FRP sheets typically does not result in the development of twice the
tensile resisting force in the sheets. As discussed in Chapter 5, this trend was also
observed for the beams in this experimental program.

Based on the bond model introduced by Maeda et al. (1997), and modified to account for
both shear and peeling stresses on the surface of I-shaped cross sections, the following
expression was introduced in Chapter 5 to predict the maximum strain in the FRP sheets
as a function of their stiffness:

\[
\varepsilon_{f,\text{max}} = \frac{0.4}{\sin \alpha_f \sqrt{E_f t_f n_f}}
\]  

(6-8)

6.4.3 Effect of CFRP Sheet Configuration on Maximum FRP Sheet Strain

Vertical FRP Sheets – Although Equation (6-8) is based on test results for beams with
diagonal FRP sheets and the maximum strains measured in diagonally oriented principal
fibres, it is recognized that only the vertical component of the force in the diagonal sheets
will produce peeling stresses. The term “\( \sin \alpha_f \)” in Equation (6-8) accounts for this
observation, and therefore, Equation (6-8) may also be used for beams with vertical FRP
sheets.
Horizontal FRP Sheet -- Although the effect of the horizontal FRP layer could not be directly quantified from the test results, it was observed during testing that the presence of the horizontal layer delayed the straightening of the FRP sheets and the corresponding reduction in the FRP contribution, $V_{f,\text{max}}$. The value for the maximum FRP strain, $\varepsilon_{f,\text{max}}$, as well as the predicted FRP contribution, $V_{f,\text{max}}$, remains unchanged with the additional horizontal layer, however the FRP contribution is sustained at this maximum level rather than dropping off. As will be discussed in Section 6.5, the presence of the horizontal layer allows the steel stirrups to yield without any reduction in $V_{f,\text{max}}$.

Clamped FRP Sheets -- Although the maximum potential of the clamped diagonal sheet configuration was not realized due to failure occurring outside of the strengthened zone, it is reasonable to conclude that the clamping scheme effectively prevents straightening of the sheets. Therefore, the maximum strain in the clamped FRP sheets can be predicted using Equation (5-4), which is based directly on the bond test results without modification due to the effects of combined shear and peeling stresses.

6.5 SHEAR RESISTANCE OF INTERNAL STEEL STIRRUPS

For the first two basic modes of failure described in Figures 6-18 (a) and (b), the average FRP strain at ultimate, $\varepsilon_{f,\text{ave}}$, is typically greater than the stirrup yield strain, $\varepsilon_{sy}$. Consequently, it is typically assumed that the steel stirrups have yielded at ultimate. For
I-shaped sections however, failure due to straightening of the FRP sheets is initiated at a lower level of FRP strain and may occur prior to yielding of the stirrups. Therefore, the effective stirrup contribution, \( V_{se} \), is based on the strain in the stirrups, \( \varepsilon_{se} \), which occurs at the initiation of failure in the FRP sheets, \( V_{f_{\text{max}}} \), and is determined using Equation (6-1).

The strain in the stirrups, \( \varepsilon_{se} \), occurring at \( V_{f_{\text{max}}} \) can be predicted based on the average strain in the FRP sheets, \( \varepsilon_{f_{\text{ave}}} \), as follows:

\[
\varepsilon_{se} = \varepsilon_{f_{\text{ave}}} \sin \alpha_f / \gamma_{fs} \tag{6-9}
\]

Where, \( \gamma_{fs} = \) ratio of the vertical component of average strain in the FRP sheets to the average strain in the steel stirrups

As discussed in Section 5.6.2, a value of 1.5 is proposed for the ratio \( \gamma_{fs} \), based on results of the beam tests conducted in this experimental program.

For beams with a mechanism in place to effectively control straightening of the vertical or diagonal FRP sheets, such as the horizontal layer of FRP sheets or the clamping scheme, test results suggest that the straightening behaviour is sufficiently controlled to allow yielding of the steel stirrups at failure.
6.6 RELIABILITY OF THE PROPOSED RATIONAL MODEL

In this section, the rational model introduced to predict the shear resistance of FRP sheets and steel stirrups in I-shaped sections is applied to each test beam using the following:

1. ACI method with a crack inclination angle of 30 degrees
2. ACI method with a crack inclination angle of 45 degrees
3. Modified sectional-truss method with a strut inclination angle of 25 degrees

The predicted shear resistance is compared with actual test results to assess the reliability of the proposed model.

6.6.1 Shear Capacity of Series S Beams

For each Series S beam, the proposed model is used to predict the contribution provided by each shear resisting component, the concrete, $V_c$, the steel stirrups, $V_{se}$, and the FRP sheets, $V_{f_{max}}$. The predicted contribution of each component is compared to the experimentally determined shear resistance of that particular component as determined in Section 6.2. The overall shear strength of the beam, $V_n$, is then calculated as the sum of the shear resisting contributions, $V_c$, $V_{se}$, and $V_{f_{max}}$, and compared to the ultimate shear failure load obtained during testing.
Table 6-1 provides a summary of the predicted values for each shear resisting component and the predicted strain values used to calculate these contributions, \( \varepsilon_{f \text{ max}}, \varepsilon_{f \text{ ave}} \) and \( \varepsilon_{se} \). It should be noted that for beams S-Diag-1 and S-Diag-2, the applied shear load at which failure was initiated, 190 kN and 195 kN respectively, was used in the comparison with the predicted nominal shear resistance.

For all of the beams, the FRP sheet contribution, \( V_{f \text{ max}} \), is predicted using Equation (6-2) where the average strain in the FRP sheets, \( \varepsilon_{f \text{ ave}} \), is determined using the strain distribution model in Equation (6-7) and the maximum FRP strain, \( \varepsilon_{f \text{ max}} \), predicted using Equation (6-8).

The maximum strain in the diagonal sheets at the initiation of failure is predicted using Equation (6-8). Since the maximum strain depends on the stiffness of the FRP sheets, the maximum strain for the double layer of sheets on beam S-Diag-2 is lower than the maximum strain for the single layer of FRP sheets on beams S-Diag-1 and S-Diag-H, as indicated in Table 6-1. In spite of the lower maximum strain value, due to the increased thickness of the double layer sheets, the predicted FRP sheet contribution, \( V_{f \text{ max}} \), is slightly higher for beam S-Diag-2, as shown in Figure 6-19 (a), when compared to beams S-Diag-1 and S-Diag-H, as shown in Figure 6-19 (b).

For beam S-Diag-CL, straightening of the diagonal sheets was effectively controlled by the clamping scheme, and the maximum strain in the FRP sheets is predicted using
Equation (5-4), which is based on bond test results and has not been modified to account for the effects of peeling stresses. The predicted FRP contribution, $V_{f \text{ max}}$, is shown in Figure 6-19 (c) for beam S-Diag-CL, and is compared to the contribution determined experimentally.

The shear resistance provided by the steel stirrups, $V_{se}$, is calculated using Equation (6-1) and is compared with the stirrup contribution determined experimentally, as shown in Figures 6-20 (a), (b) and (c). For the beams shown in Figures 6-20 (a) and (b), the average strain in the stirrups at the initiation of failure, $\varepsilon_{se}$, is calculated using the average FRP strain, $\varepsilon_{f \text{ ave}}$, and the relationship between the FRP strain and steel strain provided in Equation (6-9). However, for the beams shown in Figure 6-20 (c), the predicted stirrup resistance is based on yielding of the stirrups.

Since the maximum FRP strain is different for the double and single layer diagonal sheets, it follows that the effective stirrup strain and the stirrup contribution are different for beams S-Diag-2 and S-Diag-1, as shown in Figures 6-20 (a) and (b).

The other Series-S beams with diagonal FRP sheets, beams S-Diag-H and S-Diag-CL, include mechanisms to control straightening of the sheets, and therefore the yield strain can be used for the effective stirrup strain, $\varepsilon_{se}$. The experimentally determined stress-strain curves for the steel stirrups were presented in Chapter 4, and are used to calculate the stress in the steel stirrups based on the strain. Figure 6-20 (c) shows the stirrup contribution, $V_s$, for the Series S beams where the maximum recommended strain of
0.004 is used for the effective stirrup strain, $\varepsilon_{sc}$.

In Figure 6-21, the concrete contributions predicted using the ACI method and the modified sectional-truss method are compared to the concrete contributions determined experimentally. As discussed in Section 6.2, the concrete contribution to shear resistance is determined experimentally by subtracting the total contribution provided by the shear reinforcement from the applied shear force. The modified truss method predicts quite low values for the concrete contribution, however the shear reinforcement contributions predicted using the lower compressive strut inclination angle of 25 degrees are greater for this shear model, as shown in Figures 6-19 (a) through (c) and 6-20 (a) through (c).

Figure 6-22 shows the ratio of the overall shear capacity of the beams as determined experimentally versus as predicted using the model. The shear capacity of each beam is predicted using the three approaches described above and incorporating the rational model to predict the shear resistance of FRP sheets and steel stirrups in I-shaped sections. As shown in Figure 6-22, the use of the 30 degree crack inclination angle and the ACI method, provides the most accurate prediction. By comparison, the 45 degree truss model is consistently too conservative. The modified sectional-truss model provides a predicted shear capacity between the other two approaches, however, the relative inaccuracy of this method in predicting the individual shear resisting contributions is a concern.
6.6.2 Shear Capacity of Series B Beams

For the Series B beams, the FRP sheet contribution, $V_{f_{\text{max}}}$, is determined using the same procedure described for the Series S beams. The stirrup contribution to shear resistance, $V_{se}$, is also determined using the same procedure described for the Series S beams. However, for the beams where a horizontal sheet delays the straightening of the vertical or diagonal sheets and the potential exists for the stirrups to yield, the effect of the bent-legged stirrup shape must be considered. Due to the shape of the stirrup, failure may occur due to straightening of the stirrup legs prior to yielding of the stirrups.

Since failure of beam B-Control was completely controlled by straightening of the bent-legged stirrups, the average strain in the stirrups at which stirrup straightening typically occurs, $\varepsilon_{se^*}$, is determined using the test results for beam B-Control. The stirrup contribution, $V_s$, is determined by subtracting the predicted concrete contribution from the applied shear load at failure. Substituting the stirrup contribution, $V_s$, into Equation (6-1), the average strain in the stirrups at straightening, $\varepsilon_{se^*}$, is determined.

Using the procedure described above, a value of 0.00185 was obtained for $\varepsilon_{se^*}$ and is used as the limiting average strain for the bent-legged stirrups. For beam B-CL, where the clamping scheme effectively controlled straightening of the bent-legged stirrups, the typical limiting strain of 0.004 is used.
Figure 6-23 shows the ratio of the overall shear capacity of the beams as determined experimentally versus as predicted using the model. As expected, the use of the 30 degree crack inclination angle and the ACI method, provides the most accurate prediction. Similar to the predictions made for the Series S beams, the 45 degree truss model is consistently too conservative, and the modified sectional-truss model predicts shear capacities lying between the other two approaches. Table 6-2 provides a summary of the predicted values for each shear resisting component and the predicted strain values used to calculate these contributions, $\varepsilon_{f,\text{max}}$, $\varepsilon_{f,\text{ave}}$ and $\varepsilon_{se}$, for the Series B beams.
Table 6-1  Predicted Shear Capacity: Series S Beams

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*Concrete contribution based on Modified Sectional Truss Method (Ramirez and Breen 1991)*
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*Concrete contribution based on Modified Sectional Truss Method (Ramirez and Breen 1991)
Figure 6-1 Measurement of Strain Distribution in FRP Sheets and Steel Stirrups

Figure 6-2 Contribution to Shear Resistance Provided by FRP Sheets
Figure 6-3  Shear Resisting Components for Beam S-Diag-1

Figure 6-4  Shear Resisting Components for Beam S-Diag-2
Figure 6-5  Shear Resisting Components for Beam S-Diag-H

Figure 6-6  Shear Resisting Components for Beam S-Diag-CL
Figure 6-7  Shear Resisting Forces in FRP Sheets: Series S Beams

Figure 6-8  Shear Resisting Forces in Steel Stirrups: Series S Beams
Figure 6-9  Shear Resisting Forces in Concrete: Series S Beams
Figure 6-10  Shear Cracking Load Prediction and Test Results: Beams B-Control and B-CL

Figure 6-11  Shear Cracking Load Prediction and Test Results: Beams B-Vert-H and B-Vert100
Figure 6-12  Shear Cracking Load Prediction and Test Results: Beams B-Diag20 and B-Vert20

Figure 6-13  Shear Cracking Load Prediction and Test Results: Beams B-Diag-H and B-Diag100
Figure 6-14  Shear Cracking Load Prediction and Test Results: Beams S-Control and S-NoFRP

Figure 6-15  Shear Cracking Load Prediction and Test Results: Beams S-Diag-1 and S-Diag-H
Figure 6-16  Shear Cracking Load Prediction and Test Results: Beams S-Diag-2 and S-Diag-CL
Figure 6-17 (a) Angle of Principal Strains Just Prior to Flexural Cracking: Series B

Figure 6-17 (b) Angle of Principal Strains Just Prior to Flexural Cracking: Series S
Figure 6-18 (a) Basic Failure Mode: Rupture of FRP Sheets

Figure 6-18 (b) Basic Failure Mode: Bond Failure Due to Shear

Figure 6-18 (c) Basic Failure Mode: Bond Failure Due to Peeling and Shear
Figure 6-19 (a) Predicted vs Actual FRP Sheet Shear Contribution: Double Layer

Figure 6-19 (b) Predicted vs Actual FRP Sheet Shear Contribution: Single Layer
Figure 6-19 (c) Predicted vs Actual FRP Sheet Shear Contribution: Clamped
Figure 6-20 (a) Predicted vs Actual Stirrup Shear Contribution: Double Layer FRP

Figure 6-20 (b) Predicted vs Actual Stirrup Shear Contribution: Single Layer FRP
Figure 6-20 (c) Predicted vs Actual Stirrup Shear Contribution: Stirrups Yield

Figure 6-21 Predicted vs Actual Concrete Contribution to Shear Resistance
Figure 6-22 Reliability of Proposed Model: Series S Beams

Figure 6-23 Reliability of Proposed Model: Series B Beams
7.1 GENERAL

A rational model was introduced in Chapter 6 to predict the shear resistance provided by FRP sheets externally bonded to I-shaped concrete girders. For I-shaped sections, the concrete substrate is subjected to both peeling and shear stresses and failure is initiated by straightening of the CFRP sheets. The shear resistance provided by the FRP sheets, $V_{f_{\text{max}}}$, is calculated based on the maximum strain developed in FRP sheets subjected to both shear and peeling stresses and the FRP strain distribution model introduced for I-shaped sections.

In Chapter 6, it was also suggested that, depending upon the configuration of the FRP sheets and the corresponding mode of failure, the internal steel stirrups may not reach yield prior to failure. The stirrup contribution to shear resistance is therefore based on the load sharing relationship between the FRP sheets and the steel stirrups, and the predicted mode of failure.
In Chapter 6, it was shown that the expression for predicting maximum FRP strain, the FRP strain distribution model, and the load sharing relationship between the FRP sheets and the steel stirrups can be applied to both the traditional ACI approach to shear design and the modified sectional-truss model. It was shown that the most accurate shear resistance prediction is obtained using the ACI approach, but with a more realistic prediction for the shear crack angle based on the angle of principal compressive stresses at cracking.

The versatility of the proposed rational model is demonstrated further in this chapter, by applying the expression for maximum FRP strain, the FRP strain distribution model, and the load sharing relationship between the FRP sheets and the steel stirrups to the compression field theory and the modified compression field theory. The shear capacity of each beam tested in this experimental program is predicted using the compression field theories in combination with the proposed rational model, and the predictions are compared with the test results.

7.2 COMPRESSION FIELD THEORY

The compression field theory was described in Section 3.2.4, and the three equilibrium equations, two compatibility equations and the stress-strain relationship for diagonally cracked concrete in compression were presented.
In order to account for the presence of externally bonded FRP sheets, the three equilibrium equations must be modified as will be described in Section 7.2.1.

As mentioned in Chapter 3, the equilibrium and compatibility requirements are interrelated by the stress-strain relationships for each material. Although the stress-strain curve for the FRP sheets is a simple linearly elastic relationship, the type and configuration of the FRP sheets will determine the mode of failure and must be considered when determining the maximum stress in the FRP sheets at failure. As will be discussed in Section 7.2.2, the load sharing relationship between the FRP sheets and the steel stirrups must be used to relate the strain in these materials at ultimate to the average vertical strain, $\varepsilon_y$, used in the compatibility equations.

In Section 7.2.3, the solution procedure used to apply the compression field theory to the beams tested in this experimental program is described, and a comparison of the predictions made using the compression field theory with the results observed during testing is provided.

7.2.1 Equilibrium Equations

As described in Section 3.2.4, the compression field theory is formulated by considering equilibrium of a beam cross section subjected to shear only, as shown in Figures 3-7 (a) and (b). The presence of externally bonded FRP sheets can be accounted for as shown in Figures 7-1 (a) and (b).
By considering vertical equilibrium of the cross section shown in Figure 7-1 (a), the following expression is derived for the applied shear force:

\[ V = f_2 b_w d_v \cos \theta \sin \theta + f_r A_r d_r \cos \alpha_r \frac{d_r}{s_r} \quad (7-1\ a) \]

Where \[ A_r = 2 n_r t_r w_r \]

Based on equilibrium in the horizontal direction, the following expression for the additional tensile force due to shear, \( N_v \), is derived:

\[ N_v = f_2 \cos^2 \theta b_w d_v - f_r A_r d_r \cot \alpha_r \cos \alpha_r \frac{d_r}{s_r} \quad (7-1\ b) \]

The force in an individual stirrup is determined using the following expression, which is based on vertical equilibrium of the portion of the beam shown in Figure 7-1 (b):

\[ f_{sv} A_{sv} = f_2 b_w s \sin^2 \theta - f_r A_r \frac{s \sin \alpha_r}{s_r} \quad (7-1\ c) \]

In addition to the three equilibrium equations given above, the two compatibility equations described in Chapter 3, Equations (3-7 a) and (3-7 b), are used to determine the response of the beam. The stress-strain relationship for diagonally cracked concrete in compression, provided in Equations (3-8 a) and (3-8 b), is also used.
7.2.2 Applying the Rational Model for FRP Sheets on I-Shaped Sections

In Chapter 3, the typical failure mode was described as yielding of the steel stirrups and simultaneous yielding of the longitudinal reinforcement. For I-shaped cross sections strengthened with FRP sheets however, failure is initiated by straightening of the CFRP sheets, which may occur prior to yielding of the stirrups. Failure is therefore controlled by the maximum strain that can be developed in the FRP sheets, \( \varepsilon_{r, \text{max}} \), which is predicted using Equation (6-8). The average strain in the sheets is determined based on the FRP strain distribution model for I-shaped sections and using Equation (6-7).

During testing, it was observed that the average strain in the FRP sheets was approximately 1.5 times higher than the average strain in the steel stirrups. This phenomenon was discussed in Section 5.6, and is attributed to the superior bond performance of the thin and well-distributed FRP sheets. Prior to straightening of the FRP sheets, the sheets control the overall crack width and the average strain in the web. The average vertical strain, \( \varepsilon_y \), is therefore based on the vertical component of the FRP sheet strain rather than the stirrup strain. The stirrup strain is calculated using the load sharing relationship provided in Equation (6-9). Figure 7-2 (a) illustrates the proposed relationship between the vertical component of the FRP sheet strain, \( (\sin \alpha_r) \varepsilon_{r, \text{ave}} \), the stirrup strain, \( \varepsilon_{se} \), and the average vertical strain in the web, \( \varepsilon_y \).
As observed during testing, failure is initiated by straightening of the FRP sheets, after which the FRP strain decreases, and the stirrup strain rapidly increases. For beams with vertical or diagonal sheets only, the initiation of sheet straightening represents failure of the beam, as shown in Figure 7-2 (a).

For beams where a horizontal sheet is provided to control the straightening behaviour of the vertical or diagonal sheets, a constant level of average strain is maintained in the FRP sheets, while the stirrup strain increases rapidly and failure occurs due to yielding or straightening of the stirrups. The rapid increase in stirrup strain occurring after the FRP sheets begin to straighten, suggests that the overall crack width is controlled by the stirrups at this stage. The average vertical strain in the web, $\varepsilon_y$, is therefore equivalent to the strain in the stirrups at this stage, as shown in Figure 7-2 (b).

The straightening behaviour of the FRP sheets is effectively controlled by the clamping scheme applied to beam S-Diag-CL, and the FRP sheets continue to control the overall crack width and average strain in the web until failure, as shown in Figure 7-2 (c).

7.2.3 Comparison of Predictions vs Test Results

In this section, the solution procedure used to apply the Compression Field Theory to the test beams is described, and the predictions are compared with the test results.
As described in the previous section, the mode of failure and the average vertical strain in the web, $\varepsilon_y$, can be determined based on the configuration of the FRP sheets and using the proposed rational model. A value for the strut inclination angle, $\theta$, is then assumed, as well as a value for the principal compressive strain $\varepsilon_2$. Using compatibility Equations (3-7 a) and (3-7 b), the principal tensile strain, $\varepsilon_1$, and the longitudinal strain, $\varepsilon_x$, are determined.

The stress in the compressive struts, $f_2$, is calculated using Equilibrium Equation (7-1 c) and considering the stress in the FRP sheets and steel stirrups at failure. Using Equation (3-8 a), and considering the principal tensile strain, $\varepsilon_1$, calculated above, the maximum compressive stress allowed in the inclined struts, $f_{2\text{ max}}$, is evaluated. The stress in the compressive struts, $f_2$, is then determined based on the assumed value for the strain, $\varepsilon_2$, and using the stress-strain relationship in Equation (3-8 b). If the value for the stress, $f_2$, based on the strain $\varepsilon_2$ is not equivalent to the stress based on equilibrium, then another value for the strain $\varepsilon_2$ is selected.

Equilibrium Equations (7-1 a) and (7-1 b) are used to determine the shear force, $V$, and the longitudinal force due to shear, $N_v$. The longitudinal strain due to shear, $\varepsilon_{xv}$, is calculated using $N_v$ as follows:

$$\varepsilon_{xv} = \frac{0.5 N_v}{A_{ps} E_{ps} + A_s E_s + A_{cfs} E_{cfs}} \quad (7-2)$$
The longitudinal strains due to applied moment, $\varepsilon_{xm}$, and prestressing forces, $\varepsilon_{xps}$, are determined using the program RESPONSE (Collins and Mitchell 1997) and are added to the longitudinal strain due to shear, resulting in the total longitudinal strain, $\varepsilon_x^*$, as follows:

$$
\varepsilon_x^* = \varepsilon_{xv} + \varepsilon_{xm} - \varepsilon_{xps}
$$

(7-3)

The analysis is conducted at the centroid of the uncracked section, slightly above the mid-height of the web. If the analysis is performed lower on the cross section, and the effects of applied moment and prestressing are considered, the result is an increase in the longitudinal strain, $\varepsilon_x$, and a decrease in the predicted shear capacity, $V$.

The total longitudinal strain determined using Equation (7-3) and considering the combined effect of shear, applied moment and prestressing, $\varepsilon_x^*$, is compared to the longitudinal strain calculated previously using the compatibility equations, $\varepsilon_x$. If the two longitudinal strain values, $\varepsilon_x$ and $\varepsilon_x^*$, are not equivalent, then the assumed value for the strut inclination angle, $\theta$, is varied, and the procedure begins again with another iteration.

The ratio of the shear capacity obtained during testing, $V_{test}$, to the shear capacity predicted using the procedure described above, $V_{eh}$, is shown in Figure 7-3 (a) for the Series B beams, and in Figure 7-3 (b) for the Series S beams.
As illustrated in Figures 7-3 (a) and (b), the Compression Field Theory is quite conservative for the beams tested in this experimental program. The shear capacity predicted using the Compression Field Theory is typically over 50% greater than the shear capacity obtained during testing. It is interesting to note, however, that the strut inclination angle, $\theta$, predicted using the Compression Field Theory, as shown in Figures 7-4 (a) and (b), is similar to the shear crack angle of 30 degrees that was observed during testing.
7.3 MODIFIED COMPRESSION FIELD THEORY

As discussed in Chapter 3, the addition of a concrete contribution is intended to account for some of the additional shear resisting mechanisms, which are not accounted for in the traditional 45 degree truss analogy. By considering the beneficial effect of tensile stresses in the cracked concrete, the Modified Compression Field Theory accounts for some of these additional shear resisting mechanisms. In this section, the Modified Compression Field Theory is used to predict the shear capacity of the beams tested in this Experimental Program.

7.3.1 Equilibrium Equations

Similar to the approach described in Section 7.2.1, the three equilibrium equations can be modified to account for both the tensile stresses in the concrete, \( f_t \), and the presence of FRP sheets by considering equilibrium of the portion of the beam shown in Figures 7-5 (a) and (b). Based on vertical equilibrium of the cross section shown in Figure 7-5 (a), the following expression is derived for the applied shear force, \( V \):

\[
V = (f_1 + f_2) b_w d_v \cos \theta \sin \theta + f_r A_r d_r \cos \alpha_f \frac{s_r}{s_f} \tag{7-4a}
\]

Based on equilibrium in the horizontal direction, the following expression for the additional tensile force, \( N_v \), is derived:
\[ N_v = (f_2 \cos^2 \theta - f_1 \sin^2 \theta) bd_v - f_t A_f \frac{d_f \cot \alpha_f \cos \alpha_f}{s_f} \quad (7-4 \, b) \]

The following expression is based on vertical equilibrium of the portion of the beam shown in Figure 7-5 (b):

\[ f_{sv} A_{sv} = (f_2 \sin^2 \theta - f_1 \cos^2 \theta) b_w s - f_t A_f \frac{s \sin \alpha_f}{s_f} \quad (7-4 \, c) \]

The compatibility Equations (3-7 a) and (3-7 b), the stress-strain relationship for diagonally cracked concrete in compression provided in Equations (3-8 a) and (3-8 b), and the stress-strain relationship for concrete in tension provided in Equation (3-11) are used in addition to the three equilibrium equations shown above.

By considering the difference in the local stresses at the crack when compared to the local stresses between the cracks, and including the effect of the presence of FRP sheets, an equation similar to Equation (3-11) can be derived. In order to simplify the analysis, the same conservative assumption described in Section 3.2.4 is made. The stirrup strain at the crack and the stirrup strain between the cracks are similar and are therefore averaged. Similarly, the difference between the FRP strain at the crack and the FRP strain between the cracks are averaged. Therefore, the limit for the tensile stress \( f_t \)
provided in Equation (3-12 b) is used, and the stress-strain relationship for concrete in tension shown in Figure 3-12 applies.

7.3.2 Comparison of Predictions versus Test Results

The procedure used to apply the Modified Compression Field Theory is similar to the procedure described in Section 7.2.3 for the Compression Field Theory. The major difference is the use of the stress-strain relationship for concrete in tension to determine the stress $f_1$ based on the strain $\varepsilon_1$, which is calculated using the compatibility equations. The tensile stress $f_1$ is then used in the Equilibrium Equations (7-4 a), (7-4 b) and (7-4 c).

Similar to the approach used for the Compression Field Theory and described previously, the Modified Compression Field Theory analysis is performed considering the total longitudinal strain, $\varepsilon_x^*$, at the centroid of the section, due to the effects of shear, applied moment and prestressing forces.

The strut inclination angle $\theta$, predicted using the Modified Compression Field Theory, is shown in Figures 7-4 (a) and (b) for the Series B and Series S beams, respectively. For all of the test beams, the predicted strut inclination angle is similar to the shear crack angle of 30 degrees that was typically observed during testing.

The ratios of the shear capacity obtained during testing, $V_{\text{test}}$, to the shear capacity
predicted using the Modified Compression Field analysis, $V_{mef}$, are shown in Figures 7-3 (a) and (b) for the Series B and Series S beams, respectively. As illustrated, the predictions made using the Modified Compression Field Theory are more accurate than those made using the Compression Field Theory.
Figure 7-1 (a) Beam Cross-Section Subjected to Shear: Compression Field Theory

Figure 7-1 (b) Part of Beam Cross-Section at Bottom of Single Stirrup
Figure 7-2 (a) Vertical Strain vs FRP Strain or Stirrup Strain

Figure 7-2(b) Vertical Strain vs FRP or Stirrup Strain: Beams with Horizontal Sheet

Figure 7-2 (c) Vertical Strain vs FRP Strain or Stirrup Strain: Beams with Clamping
Figure 7-3 (a) Series B Test Results vs Predicted Shear Capacity: Compression Field Theories

Figure 7-3 (b) Series S Test Results vs Predicted Shear Capacity: Compression Field Theories
Figure 7-4 (a) Series B Strut Inclination Angles: Compression Field Theories

Figure 7-4 (b) Series S Strut Inclination Angles: Compression Field Theories
Figure 7-5 (a) Beam Cross-Section Subjected to Shear: Modified Compression Field Theory

Figure 7-5 (b) Part of Beam Cross-Section at Bottom of Single Stirrup: Modified Compression Field Theory
8.1 SUMMARY

The use of externally bonded CFRP sheets for shear strengthening of I-shaped prestressed concrete girders was examined by conducting both an experimental investigation and an analytical study. Based on the findings, a rational model is proposed to predict the shear capacity of strengthened girders.

Seven I-shaped prestressed concrete girders were strengthened using three different types of CFRP sheets for ten different CFRP sheet configurations. The beams were tested to failure at each end, to examine their behaviour and determine the most efficient strengthening scheme. The ten metre long beams are 1:3.5 scaled models of the I-shaped prestressed concrete AASHTO girders used for the Maryland Bridge in Winnipeg, Manitoba, Canada. Four of the test beams were reinforced using bent-legged stirrups with a shape identical to those used in the Maryland Bridge girders. The remaining three beams were reinforced with the more commonly used straight-legged stirrup shape, in order to extend the applicability of the experimental results.
Since the bond between the CFRP sheets and the concrete is a critical component of this strengthening method, fifteen smaller tension-type bond specimens were tested. Six rectangular bond specimens were tested, as well as nine single-flanged bond specimens designed to simulate the bottom tension flange of an I-shaped AASHTO bridge girder. The maximum strain developed in the CFRP sheets, the effective bond length and the load sharing relationship between the CFRP sheets and internal steel bars were examined.

Based on an analysis of the test results, a rational model is introduced to predict the behaviour and shear capacity of I-shaped prestressed concrete girders strengthened with externally bonded CFRP sheets. As observed during testing, for I-shaped cross-sections, failure is initiated by straightening of the CFRP sheets due to an outward force subjecting the concrete substrate to both peeling and shear stresses. The shear resistance provided by the CFRP sheets is based on the maximum FRP contribution, which occurs just prior to straightening of the CFRP sheets.

The shear resistance of the CFRP sheets is determined using an expression to predict the maximum strain developed in the sheets based on their stiffness, and an FRP strain distribution model introduced for I-shaped sections. The mode of failure and the average strain in the FRP sheets at failure depends on the type and configuration of CFRP sheets used as well as the configuration of the concrete cross-section. The steel stirrup contribution is calculated using the load sharing relationship established between the FRP sheets and steel stirrups, and is based on the strain in the FRP sheets at failure.
Design guidelines are proposed for the use of CFRP sheets for shear strengthening of I-shaped prestressed concrete girders, based on the findings of the experimental investigation and the analytical study. As demonstrated in the analytical study, the design guidelines and proposed rational model can be applied to several existing shear design approaches.

8.2 CONCLUSIONS

This study has shown that externally bonded CFRP sheets are an effective solution for the shear strengthening of I-shaped prestressed concrete girders. The following summarizes the findings of this investigation:

1. Due to the bent-legged shape of the stirrups used in the original Maryland Bridge girders, an outward force is created under increasing tensile force in the stirrups. The outward force resultant causes spalling off of the concrete cover, followed by straightening of the stirrups and premature failure. CFRP sheets are effective in reducing the tensile force in the stirrups under the same level of applied shear load.

2. Due to the shape of the concrete cross-section, the tensile force developed in the CFRP sheets subjects the concrete substrate to both shear and peeling stresses. Straightening of the CFRP sheets was observed on the lower part of the web due to the outward force resultant in the sheets.
3. The clamping scheme was effective in controlling the outward force in both the bent-legged stirrups and the CFRP sheets. Beam B-CL, with clamped bent-legged stirrups, achieved a 27% increase in ultimate shear capacity when compared to beam B-Control. As a result of the clamping scheme, higher stirrup forces were distributed to more stirrups within the shear span, allowing all of the stirrups crossing the shear crack to yield. For beam S-Diag-CL, with straight-legged stirrups and clamped diagonal CFRP sheets, the clamping scheme was so effective that failure occurred outside of the strengthened zone.

4. The premature failure of flexural strengthening schemes using CFRP strips, in zones of high shear forces, can be delayed by the use of CFRP sheets applied on top of the CFRP strip on the underside of the beam, and continuing over the height of the cross-section.

5. Load transfer between the CFRP sheets and the concrete is very localized, an indication of good bond performance. Failure typically occurs within the concrete substrate, rather than by debonding of the CFRP sheets.

6. The use of the hydro-blasting surface preparation technique has been shown to provide excellent bond performance with more localized load transfer in both the test beams and the bond specimens. The maximum strain developed in the CFRP sheets may be increased by up to 9% with the use of hydro-blasting as a surface preparation technique.
7. The maximum strain developed in externally bonded CFRP sheets depends on the stiffness of the CFRP sheets, and can be predicted based on the concept of a constant strain gradient and an effective bond length. The bond strength prediction model available in the literature was found to accurately predict the performance of the rectangular bond specimens. For I-shaped cross-sections such as the test beams and the single-flanged bond specimens, the model is modified to account for the effect of peeling stresses in combination with shear stresses.

8. The application of a second layer of diagonal CFRP sheets does not result in a proportional increase in the shear resistance provided by the sheets. This finding is in agreement with the bond strength prediction model provided in the literature.

9. Due to the superior bond performance of the CFRP sheets, when compared to the load transfer properties of the steel bars, the strain developed in the CFRP sheets may exceed the strain in the steel bars over much of a cracked concrete member. The load sharing relationship based on test beam and bond specimen results was found to accurately predict the shear resistance provided by the steel stirrups.
10. Diagonal CFRP sheets are the most efficient configuration in reducing the tensile force in the stirrups at the same level of applied shear load.

11. The application of a horizontal layer of CFRP sheets on top of diagonal or vertical CFRP sheets was found to increase the shear capacity to an even greater degree than diagonal or vertical sheets alone. While the horizontal layer does not completely prevent straightening of the CFRP sheets, the straightening behaviour is controlled enough to maintain the maximum CFRP shear contribution, allowing the stirrups to reach yield at ultimate.

12. The proposed rational model is based on an expression to predict the maximum strain developed in the CFRP sheets, the CFRP strain distribution model introduced for I-shaped cross-sections, and the load sharing relationship established between the CFRP sheets and the steel stirrups. The versatility of the proposed model was demonstrated by applying the model to various existing shear prediction models in order to account for the presence of CFRP sheets on I-shaped prestressed concrete beams.

13. The reliability of the proposed model was found to depend mainly on the accuracy of the existing shear prediction model used. If the shear prediction model allows for the use of a lower strut inclination angle, $\theta$, as is typically observed for prestressed beams, then the CFRP shear contribution and the steel stirrup contribution are
consistently accurate. The greatest variability between the shear prediction models used lies in the prediction of the concrete contribution to shear resistance.

14. The most accurate prediction for the shear capacity of the beams tested in this experimental program, was obtained using the traditional ACI approach, but with a more realistic prediction for the shear crack angle based on the angle of principal compressive stresses at cracking.

8.3 RECOMMENDATIONS

Based on the findings of the experimental investigation and analytical study, the following design recommendations and recommendations for further study are made:

**Design Recommendations**

1. The use of diagonal CFRP sheets is recommended for shear strengthening of beams where the maximum reduction in stirrup strain is required.

2. For beams where more equivalent CFRP sheet and steel stirrup contributions are desired, vertical CFRP sheets are recommended.
3. For I-shaped prestressed concrete cross-sections, the use of a horizontal layer CFRP sheet or clamping scheme is recommended to control the outward force resultant and the straightening behaviour in the sheets.

4. The average vertical component of the CFRP strain or stirrup strain should be limited to 0.004 in order to preserve the integrity of the concrete contribution.

5. In order to avoid the design penalty in using the overly conservative 45 degree truss analogy for prestressed concrete beams, the strut inclination angle, or angle of inclined shear cracks, θ, should be based on the angle of the principal compressive stresses at cracking.

Recommendations for further study --

1. The behaviour of I-shaped concrete beams shear strengthened using CFRP sheets should be evaluated for conditions of negative moment and high shear forces typically occurring near the support of continuous members, since any flexural shear cracking will originate at the top of the cross-section under negative moment conditions.

2. Alternatives for mechanically anchoring CFRP sheets for shear strengthening applications should be considered further, in terms of performance, ease of installation and durability of the rehabilitation solution.
3. For larger cross-sections, an increase in the overall height of the cross-section and use of the same value for the effective bond length of the CFRP sheets, results in a significant increase in the effective depth of the CFRP sheets, $d_f$. The beneficial effect of applying vertical or diagonal CFRP sheets to larger cross-sections should be confirmed.
REFERENCES

ACI Committee 318, (1995) "Building Code Requirements for Reinforced Concrete and Commentary," ACI 318M-95 / ACI 318RM-95, American Concrete Institute, Detroit, Michigan.


