ABSTRACT

HEISER, MATTHEW JOHN. Shear Behavior of Reduced Unit Weight Concrete Using Lightweight Synthetic Particles. (Under the direction of Dr. Sami Rizkalla.)

The objective of this research is to study the shear behavior of reinforced concrete beams containing Lightweight Synthetic Particles (LSP). LSP is an additive that, when used, leads to reduced unit weight, enhances flowability of the fresh concrete for pumping purposes and produces durable concrete for freeze/thaw and deicing exposed conditions. It also reduces the thermal conductivity of concrete, thus reducing the energy required for the heating and cooling of buildings. Use of these specially formulated particles, in combination with normalweight aggregates, could reduce the unit weight of concrete by 10% to 20%, ranging from 120 to 130 pcf, depending on the amount of LSP used in the concrete mixture.

Nine reinforced concrete beams were constructed using No. 9 and No. 10 bars for tension and compression steel, respectively, as well as No. 3 bars in the form of closed stirrups for shear reinforcement. The main variables considered in the study include the stirrup spacing, the cured concrete unit weight and compressive strength, as well as the shear span-to-depth ratio $a/d$. Testing was performed using a single concentrated load positioned closer to one end of the beam, allowing for a total of two tests to be completed on each beam. Research findings indicate that the shear behavior of beams with LSP additive is similar to the behavior of typical normalweight concrete without inclusion of the additive. Test results confirm that ACI 318-08 can be used for the conservative design of LSP concrete members for shear without the use of the reduction factor $\lambda$ required for lightweight concrete.
Shear Behavior of Reduced Unit Weight Concrete
Using Lightweight Synthetic Particles

by
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DEDICATION

To my family who taught me that success was always achievable for those who worked hard enough to earn it. Thank you.
BIOGRAPHY

Matthew J. Heiser began studying engineering in August 2006 at North Carolina State University, Raleigh, North Carolina. While an undergraduate, Matthew gained valuable work experience interning for two civil engineering firms over the course of two summers. He also worked during the school semesters as an undergraduate research assistant for several different professors from within the department. Matthew joined the research team at the Constructed Facilities Laboratory, Raleigh, North Carolina, one semester prior to graduation in December of 2008 with a Bachelor of Science in Civil Engineering. One month later, Matthew enrolled in the Graduate Program at North Carolina State University to pursue a Master of Science in Civil Engineering with a Concentration in Structural Engineering and Mechanics. Upon completion of his Master’s he plans to continue his education and attend a top ranking university in pursuit of a Doctor of Philosophy Degree.
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1.0 INTRODUCTION

The current ACI 318-08 Building Code defines normalweight concrete as concrete containing aggregates that conform to ASTM C33 with a unit weight ranging between 135 and 160 lb/ft$^3$. ACI 318-08 also defines lightweight concrete as concrete containing only lightweight aggregates conforming to ASTM C330 with a unit weight between 90 and 115 lb/ft$^3$. The lightweight synthetic particles used in this investigation are polymer spheres with a closed cell inner structure containing air and a maximum sphere diameter of 0.025 inches. They are specially formulated for use with concrete and have the ability to disperse uniformly throughout the matrix, resulting in a reduced unit weight for both plastic and hardened concretes. These particles have a specific gravity of 0.042, and since they do not conform to either ASTM C33 or ASTM C330, they are considered as an additive. Therefore, concrete made with normalweight aggregates and LSP as an additive to reduce the unit weight is regarded as normalweight concrete with “reduced” unit weight. In order for structural engineers to use the LSP in design and benefit from the overall compounded effect of reduction in the unit weight of structural members, knowledge must be obtained about the fundamental behavior of LSP additive and how it contributes to common failure mechanisms associated with reinforced concrete members. One of the most important and potentially dangerous failure mechanisms is shear, or diagonal tension failure. When examining the shear behavior of reinforced concrete beams produced with LSP additive, a key aspect of analysis and design is the application of the reduction factor $\lambda$ which is typically used for lightweight concrete. Use of this factor essentially results in a reduced shear strength.
predicted by code equations because lightweight aggregates are weak in shear, however, this
is not the case for LSP concrete. This thesis summarizes the findings of an extensive
research program conducted to examine the compliance to ACI 318-08 and other codes for
the shear behavior of concrete members containing LSP.

1.1 Objective

The primary objective of the investigation presented in this thesis is to demonstrate
through experimental observation that ACI 318-08 and other code provisions can be used for
the shear design of structural concrete members containing LSP additive and normalweight
aggregates. The primary objective will be achieved through the following tasks:

1. Evaluate the shear behavior of reinforced concrete beams containing lightweight
   synthetic particles and varying amounts of transverse reinforcement.
2. Evaluate the effect of different parameters influencing the shear behavior.
3. Evaluate the capability of current design codes to predict the shear strength.
4. Examine the applicability of the modification factor $\lambda$ associated with lightweight
   concrete.

1.2 Scope

The overall experimental program consisted of nine large-scale reinforced concrete
beams, tested under static loading up to failure to evaluate the shear behavior of the concrete.
Each beam was tested once at each end to replicate the test results. The steel reinforcement
used had a specified minimum yield strength of 60 ksi, according to ASTM A615/A615M
specifications. The main variables considered in the investigation are the concrete unit
weight, concrete compressive strength, the amount of transverse reinforcement, and the shear span-to-depth ratio $a/d$.

Following this introduction, Chapter 2 presents some of the most relevant information published on the subject of shear behavior including the shear resistance provided by both concrete and steel, available models typically used to examine shear behavior, as well as background information on several of the most current design codes. Also included in this chapter is a brief summary of the fabrication process of LSP and the material characteristics to better understand the behavior of the LSP additive used in the investigation. Chapter 3 presents the experimental program and illustrates the details of the fabrication of all of the test specimens, the test setup, instrumentation, and the testing procedure. The results obtained from the experimental program are presented in Chapter 4 along with an extensive discussion about the program as well as a comparison of these results to the codes outlined in the literature review of Chapter 2. Finally, a summary of the research findings and recommendations for future research are presented in Chapter 5. In addition to the main body of the thesis, two appendices have been included to provide further information regarding the beam details, test results, and analysis.
2.0 LITERATURE REVIEW

2.1 Introduction

Additives and chemical admixtures have been used for years to alter the mechanical properties of normalweight concrete. With a wide range of performance enhancing capabilities, these commercially available products can be advantageous in the overall design and construction of reinforced concrete structures. Among the range of benefits that can be achieved by using such products, one that is highly advantageous is reduction of dried concrete unit weight. With the use of Lightweight Synthetic Particles (LSP) derived from expanded polystyrene (EPS) spheres, this can be achieved along with other improved performance and behavior characteristics. The LSP used for this research are produced by NOVA Chemicals Inc. and are commercially known as Elemix® additive. Use of these specially formulated particles reduces the unit weight of concrete by 10 to 20% when used in combination with normalweight aggregates and produces concrete with unit weight ranging from 120 to 130 lb/ft\(^3\) depending on the amount of the additive in the concrete mixture. The addition of LSP to a concrete mixture provides significant benefits, including better flowability and workability for fresh concrete, improvement of the thermal performance which increases the R-value and reduces the energy required for heating and cooling of buildings, in addition to the reduction of the unit weight for the hardened concrete (<http://www.elemix.com/technical.aspx>, Accessed January, 2009). The scope of this research program is to investigate the shear behavior of reinforced concrete flexural members containing LSP additive. This chapter provides an introduction to the LSP concrete
properties and also summarizes the fundamental mechanism of the shear behavior of concrete members to assess the compliance of LSP concrete as normalweight concrete for reinforced concrete structural applications.

2.2 **Lightweight Synthetic Particles (LSP)**

The Lightweight Synthetic Particles are derived from polystyrene beads that are steam expanded and specially formulated so that when incorporated into a concrete mixture, enhanced behavior and performance of the concrete can be achieved. Although they are similar to EPS in the formulation, the LSP used in this experimental investigation are different than the particles that we see in our everyday lives such as those used in the production of beverage containers, beanbag chairs, insulation for our homes, and even surfboards. Although the uses of EPS and the LSP are quite different, the manufacturing process to achieve the final product is similar.

Manufacturing of the LSP is typically completed in three steps. As seen in Figure 2.1, all of the ingredients for the particles are added into a reactor where they are heated and stirred to form the “premature” polystyrene spheres for the first step of the process. The spheres are then conveyed to large sieves and passed through screens that sort them into storage containers based on their size as shown by Step 1 in Figure 2.1. Undesired particles are separated from the batch, recycled and passed back through the reactor to repeat the process.

In the second step, the prime spheres that were not discarded are mixed with water and the desired additives, heated in a second reactor, and then stirred with the addition of a
blowing agent to form the final product as shown by Step 2 in Figure 2.1. The resulting polystyrene spheres are then processed and packaged for future use. The final step of the manufacturing process involves feeding the spheres into an expansion chamber where they are expanded to the appropriate density using steam and air. The desired density is decided based upon how the polystyrene spheres will be used. In the case of Elemix Concrete Additive to produce concrete with reduced unit weight, this process yields the smooth-skinned, closed-cell LSP shown in Figure 2.2.
Ingredients are added, then heated and stirred to form desired polymer spheres:

**Step 1**

Spheres are dried and sent to screens to separate by size.

After screening the spheres are saved in silos.

Polymer spheres, water and desired additives are heated and stirred to form final product. Blowing agent is added during this process:

**Step 2**

After hardening, the spheres are processed and packaged.

**Step 3**

Feeding into the Expansion Chamber

Some Converters Have Vacuum Chambers

Expansion to the Required Density

Discharge

Sieve Unit

Fluid Bed

Steam

Drying Air

Cellular Wheel Sluice

Figure 2.1: Manufacturing process of the LSP (Property Nova Chemicals Inc.)
2.2.1 Effective specific gravity (SG) of the LSP

Specific gravity (SG) is defined as the density or unit weight of a particular substance divided by the unit weight of water. The specific gravity of the LSP is an important material characteristic that enables the appropriate batch loadings to be formulated. There are several methods used for determining the specific gravity of the LSP, one of which involves using the gas pycnometer to first determine the volume (Hileman, 2008). The gas pycnometry method, which is based on Boyle-Mariotte’s law of volume-pressure relationship, is an effective way of determining the volume of solid particles because many
of the downfalls inherent with past volume measuring methods are eliminated, such as physiochemical reactions with water and air entrapment around adjacent particles (Tamari & Aguilar-Chávez, 2004). The type of pycnometer used in determining the volume, and thus the specific gravity, of LSP is called the constant-volume gas pycnometer, which is illustrated in Figure 2.3. The mass of the sample being tested is first measured elsewhere, typically by weighing, and the density is determining based on the mass-to-volume ratio. The constant-volume pycnometer consists of two chambers, one to hold the test sample (Sample Chamber) and one with a fixed, known internal volume (Tank). In addition to these two chambers, there is a valve to control the gas supply or vacuum (Valve “M”), and a valve that isolates the sample chamber (Valve “Z”). Pressure is measured using an absolute pressure transducer (“P”).

The sample volume is determined by first placing the sample in the chamber labeled “Sample Chamber”, then the valves are opened and the pycnometer is filled with gas. Valve “M” is then closed and the absolute pressure transducer “P” is used to measure the initial gas pressure in the pycnometer, \( P_i \). Next, valve “Z” is closed to isolate the sample chamber and valve “M” is opened, introducing some gas into the tank, or removing from it. Valve “M” is then closed again and a second gas pressure in the tank is measured, \( P_j \). Finally, valve “Z” is reopened to let gas expand from the tank to the sample chamber and the final gas pressure, \( P_f \), is measured when the gas expansion process is finished.
The equations used for determining the volume of solids in the gas pycnometer are as follows:

\[ V_s = V_c^\circ + V_i^\circ \tau \]  
\[ \tau = \frac{P_f - P_i}{P_f - P_t} \]  

It can be seen from the above equations that the estimation of the sample volume \( V_s \) is based on a linear relationship with the tank volume \( V_i^\circ \) and the coefficient \( \tau \), representing the slope of the line from the origin indicated by the sample chamber volume \( V_c^\circ \).
Although the constant-volume gas pycnometer is a good solution for determining the density of the LSP, there are several drawbacks to this method. The first is that it would be inappropriate to assume that a gas pycnometer will be used on-site where batching takes place. Secondly, there is variance in the reported specific gravity values between the gas pycnometer and the actual observed behavior of the LSP in concrete. It is believed that due to the fragile nature of the LSP, densification is experienced during the mixing process which changes the effective specific gravity. This effect is also further complicated by the presence of air voids which, in combination with the LSP, will impact the amount of concrete produced in a single batch.

Alternatively, an investigation was conducted using different concrete samples to determine the effective specific gravity of the LSP more accurately. Three different concrete batches were prepared using the same concrete mixture but with various LSP densities which were obtained during the expansion process during production. The specific gravity of the LSP was predicted using regression analysis of the data obtained by the study and the results indicate that a specific gravity of 0.0422 can be used for LSP with a target vibrated bulk density of 1.455 ± 0.05 pcf. If the effective specific gravity falls outside of this range, the following two empirical equations can be used:

Density in lb/ft$^3$:  $SG_{\text{eff}} = 0.0221\rho_{VB} + 0.0102$  \hspace{1cm} \text{Eq. 2.3}$

Density in kg/m$^3$:  $SG_{\text{eff}} = 0.0014\rho_{VB} + 0.0102$  \hspace{1cm} \text{Eq. 2.4}$
where:

\[ SG_{eff} \] is the effective specific gravity;
\[ \rho_{VB} \] is the vibrated bulk density;

2.2.2 Moisture Absorption

The moisture absorption of the LSP was tested at CTL Group (CTL Group, 2008a). A specialized test method was developed using a vacuum desiccation system that does not require high temperatures, but does require longer time to dry the material sufficiently. The test setup involved using two different samples of LSP, each submerged in water inside of a glass container with a screen insert to hold the particles in place. After soaking for 24 hours, the LSP samples were removed from the container and conditioned to a saturated surface dry (SSD) state as defined in ASTM C 128, Section 6.2.1, Note 3, Item 4. Afterwards, the SSD particles were weighed and then placed inside of a vacuum desiccation chamber maintained at a pressure of -20 in. Hg (-68 kPa). The particles were weighed periodically until the weight became constant, which indicated that they were dry. The absorption capacity was calculated based on the measured values of the samples as given in Table 2.1. Only two values are reported for the tested samples and there is no literature available for the other types of samples. Further investigation is required to verify and to confirm the reported moisture absorption capacities of the LSP.
2.3 **Properties and Mechanics of LSP concrete**

Use of the LSP as an additive to normal weight concrete has shown clear benefits to the fresh and hardened properties in addition to the reduction of the unit weight. The uniformity of the particles and their low abrasion characteristics also reduce the wear of pumping, placing and finishing equipment. Their presence enhances the flowability and workability, as the small particles act as ball bearings among the concrete constituents (<www.elemix.com>, Accessed January, 2009).

For the hardened properties, research has proven that concrete compressive strength within the structural range can be achieved using LSP concrete (Hosny, 2010). The corresponding Modulus of Elasticity can be determined using the equations provided by ACI 318-08 (ACI 318-08, 2008) as follows:

\[
E = 33w^{1.5} \sqrt{f'_c}
\]

where:

- \( E \) is the modulus of elasticity of concrete, psi;
- \( w \) is the unit weight of the concrete, lb/ft\(^3\);
- \( f'_c \) is the concrete compressive strength at 28 days, psi;
Fire and vertical tube furnace tests were conducted on the LSP by Hughes Associates, Inc. (Hughes Associates, 2007) and Intertek (Intertek, 2008-1, 2, 3, and 4) respectively, to evaluate the effect of the presence of the LSP in the concrete on its fire resistance. The tests concluded that neither smoke nor ignition was observed in the tested samples. The results indicated that the presence of the LSP in the concrete samples did not have a significant impact on the fire resistance in comparison to normal weight concrete.

The mechanical properties of LSP concrete have been investigated by many researchers who discovered that several factors could affect its behavior. One of these factors includes the cracking behavior concentrated around particles occupying space within the concrete matrix. As reported by Hileman (2008b), fracture mechanics can be used to model and predict the stress distribution within the fracture process zone of concrete containing spherical particles. LSP also has a significant effect on the concrete compressive strength as the particles occupy an amount of load-bearing material, the concrete matrix, contained within a certain volume of concrete. This effect is referred to as the “particle size effect” and several FEA simulations (Blain Hileman, 2008b) have been conducted in order to determine the effect of the LSP particle size with respect to the maximum stress concentration around the particle. The effect of the course aggregate size, strength, shape, and surface condition was also investigated and proven to have a significant effect on the overall performance of LSP concrete. A more detailed explanation on the mechanical properties of reduced unit weight concrete with LSP additive can be found in Hosny (2010).
2.4 **ACI 318 Classification of LSP concrete**

According to ACI 318-08, normalweight concrete is defined as “Concrete containing only aggregates that conform to ASTM C33”. The ACI 318 code also states in its commentary that “Normalweight concrete typically has a density (unit weight) between 135 and 160 lb/ft³, and is normally taken as 145 to 150 lb/ft³.” Lightweight concrete is defined as “Concrete containing lightweight aggregates and an equilibrium density as determined by ASTM C567, between 90 and 115 lb/ft³.” In this research, Elemix is considered as an additive because it cannot be regarded as an aggregate as defined by either ASTM C33 or ASTM C30 and is consistent with ACI 318-08 which defines admixtures as “material other than water, aggregate, or hydraulic cement, used as an ingredient of concrete and added to concrete before or during its mixing to modify its properties.” For these reasons, the concrete made with the Elemix additive using normalweight aggregates and resulting in unit weights above 115 lb/ft³ should be regarded as a normalweight concrete albeit having a lower unit weight than the conventional concrete and can be considered as a concrete with “reduced” unit weight. The design of concrete members containing the Elemix additive would be governed by the provisions of ACI 318-08 for normalweight concrete and the modification factor \( \lambda \) in the code, normally associated with lightweight concrete, should not be used for concrete with LSP additive.

2.5 **Shear Transfer Mechanisms**

For many years, shear or diagonal tension in reinforced and prestressed concrete structures has been a concern for design engineers and a popular topic for researchers due to
the sudden and brittle nature of shear failure. Interest in shear was greatly influenced by the collapses of roof frames at the Wilkins Air Force Base in Shelby, Ohio, and the Robins Air Force Base in Macon, Georgia, in 1955 and 1956, respectively. Subsequent investigations determined that the primary mode of both failures was diagonal tension due to dead load coupled with axial tension due to shrinkage (Feld & Carper, 1997; NCHRP, 2005). Since then, careful attention has been paid to the detailing of transverse and longitudinal reinforcement to ensure that shear failure does not control in the design of reinforced concrete members.

Flexural failure can be accurately predicted through the application of Bernoulli-Euler Beam Theory, which is upheld under the assumption that plane sections remain plane during bending. The design of a reinforced concrete member in flexure is performed within the regions of maximum moment. Once the nominal moment capacity of the member is determined at these locations, then the same design capacity is also adequate for other portions of the member where the moment demand is reduced. Unlike flexure, shear failure is concentrated within a finite region where internal moments, shear and sometimes axial forces are present. Beams and other concrete structures can be separated into what are generally known as “B-regions” and “D-regions”, where “B” stands for “Bernoulli” or “Beam”, and “D” generally stands for “Disturbed”. “B-regions” are present where beam theory is applicable and the assumption that plane sections remain plane can be used. “D-regions” occur where the primary shear resistance of the member is provided by the formation of struts that carry in-plane forces for the member (MacGregor & Wight, 2009).
Research has shown that the shear resistance of reinforced concrete members is consequently the summation of several internal shear transfer mechanisms (NCHRP, 2005). These mechanisms include shear in the uncracked compression zone, aggregate interlock or interface shear transfer, dowel action of the longitudinal reinforcement, residual tensile stresses across the crack, and the presence of transverse shear reinforcement. Shear in the uncracked compression zone relates to the portion of the concrete member that is uncracked and able to fully resist shear forces. This contribution is limited by the depth of the compression zone, which is ultimately controlled by the size of the member itself. Aggregate interlock is attributed to local roughness along the slip surfaces of a crack allowing the protruding aggregates to “interlock”. This mechanism is presently termed “interface shear transfer” due to the variation in concrete strength that may dictate the path of crack propagation and smoothness of the cracked plane. Dowel action is attributed to the development of vertical shear forces across the longitudinal steel reinforcement, which is dependent upon the concrete cover and the presence of transverse shear reinforcement. Figure 2.4 shows the different contributing forces to the shear resistance within a cracked section of a reinforced concrete beam.
2.5.1 Members without Web Reinforcement

Shear behavior in reinforced concrete beams without web reinforcement is controlled by the participation of several important factors. These factors are the longitudinal reinforcement, course aggregate size, concrete compressive strength, member depth, and the shear span-to-depth ratio. The amount of longitudinal steel in a reinforced concrete beam is expressed by the ratio of longitudinal steel to the effective area of concrete. This ratio, $\rho_w$, is known as the longitudinal reinforcement ratio. By increasing $\rho_w$, flexural stresses and strains are decreased along with crack widths. The shear resistance is also enhanced by the dowel action mechanism. Conversely, by decreasing $\rho_w$, dowel action is reduced and internal flexural stresses increase resulting in larger crack widths and reduction in the shear load carrying capacity. Large course aggregates provide greater shear resistance by increasing the
aggregate interlock. The effect that course aggregate size has on the shear strength has also been explored in the work of Bazant and Kim (1984) in which fracture mechanics was used to develop a theoretical model that included this parameter. As previously stated, the concrete compressive strength has a significant effect on the mechanism of aggregate interlock, coining the more current term “interface shear transfer”. When high strength concrete is used in the construction of structural members, the strength of the cement matrix may exceed the strength of the course aggregate, resulting in shear cracks that pass through the aggregate. Consequently it is believed that increasing concrete strength causes a reduction in interface shear transfer due to a relatively smooth cracked plane (NCHRP, 2005). The research conducted by Angelakos et al. (2001) confirms this and also suggests that increasing concrete cylinder strength does not necessarily result in increased shear resistance. In fact, for members without transverse shear reinforcement, the margin of safety against shear failure decreases significantly with increased cylinder compressive strength. In addition, the margin will continue to become smaller as the longitudinal reinforcement ratio decreases and member size increases (Angelakos et al., 2001).

As the depth of a reinforced concrete member without shear reinforcement increases, the shear strength of the member decreases. This phenomenon has been termed the “size effect”. Research conducted by Kani (1967) shows that for beams with varying depths and constant longitudinal reinforcement ratios, the overall shear load carrying capacity decreased as the depth of the beam increased. The field of fracture mechanics has also been used to relate beam depth to shear strength of members without transverse reinforcement. In
conventional strength-based analysis of reinforced concrete, the nominal average shear stress on a section is equal to the shear load at the section divided by the cross-sectional area. This approach does not account for the varying size of the member. For instance, a thin-webbed deep beam may have the same nominal area as a shallower beam with a wider web, resulting in the same calculated shear stress at failure. Since concrete is a quasi-brittle material, meaning it is not completely brittle or ductile, the application of linear elastic fracture mechanics (LEFM) to brittle fracture is not applicable. Hence, the need for a different form of fracture mechanics is necessary for application to reinforced concrete. Bazant & Kim (1983) developed a new approach to fracture mechanics that treats concrete fracture differently than conventional LEFM. This new approach, similar in nature to elastic-plastic fracture mechanics (EPFM), takes into account the presence of tiny microcracks densely spaced throughout the fracture process zone. The application of fracture mechanics to cracking of reinforced concrete has upheld the assumptions that there is a physical size effect in shear.

The shear span-to-depth ratio or slenderness ratio, $a/d$, is an extremely influential parameter on the shear-load carrying capacity of reinforced concrete beams. The shear span of a beam, $a$, can be easily visualized as the distance between the point of application of a single load and the distance to a simple support. The ratio of this distance to the effective depth of the beam can also be expressed as the ratio of the maximum applied moment, $M_{max}$, to the maximum applied shear, $V_{max}$ multiplied by the effective depth, $d$. Since $M_{max} = V_{max} \times a$, the shear span-to-depth ratio can be expressed mathematically by the equation $M_{max}/V_{max}d$.
= a/d. As the effective depth of a beam increases, and the shear span decreases, the average stress at failure becomes progressively larger due to the formation of a compression strut that carries the shear force from the point of the load application to the support. This effect is more commonly known as “arching action”. Experimental data collected by Kani (1964) showed that as beam dimensions remained constant, the failure mode became dependent upon a/d which was influenced by both the loading and the support conditions. The results of these tests are shown in Figure 2.5, which are plots of the maximum moment and shear force of the test specimens at failure versus the shear span-to-depth ratio. It can be observed that as the type of test specimen progresses from deep beams, where the effective depth is relatively large, to slender beams, the maximum failure moment decreases with increasing a/d.
Figure 2.5: Effect of $a/d$ on shear strength of members without transverse reinforcement

(NCHRP, 2005)

It can also be observed that for reinforced concrete beams, the value of $a/d$ has a direct impact on the type of shear cracking that occurs within the shear span. This, in turn, also relates to the type of failure exhibited by the member. For beams ranging from short and
deep to long and slender, here are examples of how the shear span-to-depth ratio affects the failure mode:

a) **Deep beams** ($a/d \leq 1.0$): Cracking occurs along the diagonal between the point of load application and the support, resulting in a compression strut that carries the shear force by means of the “arching action mechanism”. Figure 2.6 shows the concept of arching action for deep beams.

![Figure 2.6: Arching action in deep beams](image)

There are several different failure mechanisms that these beams may incur. *Anchorage failure* may occur in the end of a tension tie, depending on the tie anchorage system used in the beam. If the capacity of the diagonal compression strut is greater than the bearing capacity at the nodal zone just below the point of...
load application, then a localized bearing failure may occur here due to concrete crushing. With combined moment and shear, flexural cracks are also present within the shear span, which may result in yielding of the longitudinal tension steel or crushing of the concrete in the nodal zone, creating a tension failure. The flow of tensile and compressive stresses throughout the shear span follow trajectories along the direction of the principal stresses. Since concrete is weaker in tension than in compression, splitting occurs in the principal tensile stress direction which is orthogonal to the direction of principal compressive stress (Kani, 1964). Likewise, because of these trajectories, splitting tensile cracks form the boundary of the inclined compression strut within the short shear span region. After crack initiation, eccentricity of the compressive stress thrust line may result in tension failure or arch rib failure. Also, failure of deep beams could be caused by compression strut failure. Typical failure mechanisms for deep beams are shown in Figure 2.7.
b) **Short beams (1.0 < a/d ≤ 2.5):** Failure of short beams may occur under the influence of two different mechanisms due to an increase of a/d from 1.0, as was the case for deep beams. Splitting cracks may form along the longitudinal tension steel at the bottom of the compression strut resulting in what is known as *shear-tension failure*. Conversely, *shear-compression failure* may occur due to concrete crushing within the nodal zone at the top of the compression strut. The *shear-tension* and *shear-compression* mechanisms are shown in Figure 2.8 and Figure 2.9, respectively.
c) Intermediate or slender beams (2.5 < a/d ≤ 6.0): In slender beams a wider spectrum of cracking can be observed due to the increased shear span. Vertical flexural cracks propagate upwards from the side of maximum tensile stress. Moving along the beam as the internal moment decreases and the internal shear force increases, these flexural
cracks turn into flexure-shear cracks which begin vertically and then rotate at an angle and propagate toward the externally applied load. Close to the end support, web shear cracks can be observed in a direction orthogonal to the principal tensile stress. In I-shaped beams, web shear is more critical since the web is much narrower than the flange and carries a majority of the shear stress from the entire cross section. Examples of flexural, flexure-shear and web cracks are shown in Figure 2.10. For slender beams with a minimal $a/d$, flexure-shear cracks coalesce into a primary diagonal shear crack, causing the tension steel to yield which results in diagonal tension failure.

**Figure 2.10: Intermediate beam shear cracking ($2.5 < a/d \leq 6.0$)**

- **d)** Long beams or “very slender” beams ($a/d > 6.0$): For long beams the likelihood of shear failure occurring is very rare. Because of the high amount of internal moment
at mid-span, the predominant failure mode is flexural, long before the formation of inclined cracks near the support.

2.5.2 Web Reinforcement

Web reinforcement in concrete members is necessary to carry the shear load after initiation of shear cracking in order to ensure flexural failure. Since shear failure is extremely brittle in nature, there are generally no foretelling signs of the event to occur, making it absolutely necessary to include sufficient web reinforcement in order to provide additional ductility and development of the full flexural capacity of the member (MacGregor & Wight, 2009). Web reinforcement is provided in the form of No. 3 to No. 5 stirrups, which are typically closed and tied to the longitudinal reinforcement to provide additional concrete confinement. Research shows that both vertical and angled stirrup orientations provide sufficient anchorage and transfer of the web shear force. Bent longitudinal bars that extend into shear-critical regions have also been shown to be a form of web reinforcement, and have been used in early shear design models (Kani, 1969). It is known that stirrups do not engage until the onset of diagonal tension cracking, however one of the most relatively unknown aspects of shear analysis and design is how stirrups carry the shear force within a reinforced concrete beam. This concept will be further discussed in the next section.
2.6 Mechanisms of Shear Resistance

2.6.1 Truss Model

The truss model for the post-cracking behavior of slender beams failing in shear was first proposed in two independently published papers by the Swiss engineer Ritter and the German engineer Mörsch in 1899 and 1902, respectively. The early truss model proposed by Ritter in 1899 assumed that evenly distributed shear stresses in the effective web area were carried by both the struts formed by diagonally cracked concrete and the transverse reinforcement in the form of stirrups. Ritter’s model assumed an inclined crack angle of 45° from the horizontal axis and did not account for tension in the uncracked concrete. Mörsch later suggested different angles of crack inclination as well as the potential for cracks to cross multiple stirrups along the length of the member (NCHRP, 2005; Ramirez & Breen, 1991).

Figure 2.11: Simple 45° Truss Model (MacGregor & Wight, 2009)
An illustration of the simple 45° truss model is presented in Figure 2.11. It can be seen from Figure 2.11(a) that compressive stresses generated in the concrete diagonals push the top and bottom faces of the beam apart, while the tensile stresses generated in the vertical stirrups simultaneously pull them together. Through static equilibrium the summation of the vertical and horizontal internal force components must be equal to zero, and according to the models proposed by Ritter and Mörsch, shear failure is assumed when the stirrups yield corresponding to the shear stress represented by Eq. 2.5, where $A_v$ is the effective area of shear reinforcement and $f_y$ is taken as the yield strength of vertical stirrups (ACI 445R-99, 2009).

$$v = \frac{A_v f_y}{b_w S} = \rho_v f_y$$ \hspace{1cm} \textbf{Eq. 2.5}

This indeterminate system of internal forces can be simplified into an equivalent determinant system resembling a pin-jointed planar truss illustrated by Figure 2.11(b), where all of the stirrups crossing section $A$-$A$ are lumped together to form the tension member $b$-$c$, and the diagonal concrete strut crossing section $B$-$B$ forms the compression member $e$-$f$ (MacGregor & Wight, 2009). In order for the truss to be properly analyzed, all of the stirrups are assumed to yield. The longitudinal tension steel and uncracked compression zone are also represented in the model by the bottom tension chord and the top compression chord, respectively.

The 45° truss model has been adopted by the American design code ACI 318-08 as a simple way for designing reinforced and prestressed concrete beams for shear. The steel contribution to the shear capacity of the member calculated from the model, $V_s$, is used in
combination with a concrete contribution to the shear resistance, $V_c$, since the truss model does not directly account for shear failure mechanisms such as aggregate interlock, dowel action and shear carried by the uncracked compression zone. Generally, the 45° truss model is conservative in predicting the shear capacity of members due to the angle of crack inclination which is typically flatter than 45°.

In order to improve upon the conservative predictions of the 45° truss model, the variable angle truss model was later introduced. In this model a number of compression diagonals form a compression fan originating at the point of load application, and similarly, at the support reaction. Between these two compression fans lies a region referred to as the “compression field”. The angle of inclination $\theta$ of the compression field is determined by the number of stirrups required to resist the vertical component of the compression diagonals. Like the 45° truss model, the variable angle truss model also assumes yielding of the stirrups however lower values of $\theta$ exist, typically less than 45°.

2.6.2 Modified Compression Field Theory

The Modified Compression Field Theory (MCFT) was first presented in a paper published by Collins and Vecchio (1986) as an extension to the Compression Field Theory introduced in 1974 by Mitchell and Collins (1974). The Compression Field Theory is a theoretical model for structural concrete subjected to both diagonal tension and torsion which was influenced by the work of a German engineer, Wagner, who developed the Tension Field Theory while examining the behavior of “stressed-skin” aircraft in shear (ACI 445R-99, 2009). In the Compression Field Theory model, the shear capacity of a member is dependent
upon the crack angle, \( \theta \), deformation in the transverse reinforcement, strains in the longitudinal reinforcement, and stresses in the diagonally cracked concrete. The MCFT is similar to its predecessor, the Compression Field Theory, except that it accounts for tensile stresses in cracked concrete.

Equilibrium equations that relate the concrete and reinforcement stresses to externally applied loads are expressed in terms of average stresses over a finite length greater than the crack spacing. It is understood that failure may not be due to average stresses but rather localized at a crack. This is taken into account with the MCFT when checking equilibrium conditions at a crack, as a complex crack pattern is idealized as a series of parallel cracks all oriented at an angle \( \theta \) from the longitudinal reinforcement and spaced equally apart by a distance \( s_\theta \). Compatibility requires that all reinforcement is assumed to be anchored and fully bonded to the concrete thereby equating any concrete deformation to an equivalent steel deformation. If the two normal strain components \( \varepsilon_x \) and \( \varepsilon_y \) are known in conjunction with the shear strain \( \gamma_{xy} \) then, from the geometry of Mohr’s circle, any other strain component can be determined. Two strains of particular importance obtained from geometry are the principal tensile strain \( \varepsilon_1 \) and the principal compressive strain \( \varepsilon_2 \). Since these two strains are orthogonal, the angle of the principal compressive strain \( \theta \) can be determined.
\[
\gamma_{xy} = \frac{2(\varepsilon_x - \varepsilon_2)}{\tan \theta}
\]
\[\text{Eq. 2.6}\]

\[
\varepsilon_x + \varepsilon_y = \varepsilon_1 + \varepsilon_2
\]
\[\text{Eq. 2.7}\]

\[
tan^2 \theta = \frac{\varepsilon_x - \varepsilon_2}{\varepsilon_y - \varepsilon_2} = \frac{\varepsilon_1 - \varepsilon_y}{\varepsilon_1 - \varepsilon_x} = \frac{\varepsilon_1 - \varepsilon_y}{\varepsilon_y - \varepsilon_2} = \frac{\varepsilon_x - \varepsilon_2}{\varepsilon_1 - \varepsilon_x}
\]
\[\text{Eq. 2.8}\]

(a) Average strains in a cracked element  
(b) Mohr’s Circle for average strains

Figure 2.12: Compatibility conditions for a cracked element (Collins & Vecchio, 1986)
Forces applied to the reinforced concrete element shown in Figure 2.13 are resisted by stresses in both the concrete and the reinforcement. For equilibrium, the requirement that the horizontal and vertical forces must sum to zero can be expressed by the following equations:

\[ \int_A f_x dA = \int_{A_c} f_{cx} dA_c + \int_{A_s} f_{sx} dA_s \]  \hspace{1cm} \text{Eq. 2.9} \\
\[ \int_A f_y dA = \int_{A_c} f_{cy} dA_c + \int_{A_s} f_{sy} dA_s \]  \hspace{1cm} \text{Eq. 2.10} 

For Eq. 2.9, equilibrium is observed as the total stress in the x-direction \( f_x \) is equal to the summation of both the concrete stress \( f_{cx} \) and the reinforcement stress \( f_{sx} \). A similar relationship exists for vertical stresses in the y-direction, as shown in Eq. 2.10. After the assumption that the cross-sectional area of concrete occupied by reinforcement is exiguous, the following equilibrium equations can be derived:
The assumption can then be made that \( \nu_{cx} = \nu_{cy} = \nu_{cxy} \). The concrete element shown in Figure 2.14 will resist concrete shear stress \( \nu_{cxy} \), vertical concrete stress \( f_{cy} \) and horizontal concrete stress \( f_{cx} \). Taken with respect to the crack angle \( \theta_c \), these stresses are summed to form the principal tensile stress \( f_{c1} \) and the principal compressive stress \( f_{c2} \), illustrated in Figure 2.14. Using Mohr’s circle for the average concrete stresses shown in Figure 2.14(c), the following relationships can be derived:

\[
\begin{align*}
    f_{cx} &= f_{c1} - \frac{\nu_{cxy}}{\tan \theta_c} \quad \text{Eq. 2.15} \\
    f_{cy} &= f_{c1} - \nu_{cxy} \cdot \tan \theta_c \quad \text{Eq. 2.16} \\
    f_{c2} &= f_{c1} - \nu_{cxy} \cdot \left( \tan \theta_c + \frac{1}{\tan \theta_c} \right) \quad \text{Eq. 2.17}
\end{align*}
\]
Constitutive models for both the concrete and the reinforcement are used to relate the average stress-average strain values obtained from the equilibrium equations used in the MCFT model. For simplicity, these values are assumed to be completely independent of each other.

The axial stress in the reinforcement is assumed to be singularly independent upon the axial strain, adopting a simple bi-linear model with a yield plateau for both the x and y directions.

To relate the principal strain to the principal stress in the concrete, it is assumed that the angle of principal stress $\theta_c$ is equal to the angle of principal strain $\theta$. Also, it was found by observation that the principal concrete compressive stress $f_{c2}$ is dependent upon both the principal tensile strain $\varepsilon_1$ and the principal compressive strain $\varepsilon_2$. The suggested constitutive model for the concrete is:
\[ f_{c2} = f_{c2\text{max}} \cdot \left[ 2 \left( \frac{\varepsilon_2}{\varepsilon'_c} \right) - \left( \frac{\varepsilon_2}{\varepsilon'_c} \right)^2 \right] \quad \text{Eq. 2.18} \]

where:

\[ \frac{f_{c2\text{max}}}{f'_c} = \frac{1}{0.8 - 0.34 \frac{\varepsilon_1}{\varepsilon'_c}} \leq 1.0 \quad \text{Eq. 2.19} \]

During a typical compression test of a concrete cylinder, the strain corresponding to the peak stress is generally equal to -0.002, as represented by \( \varepsilon'_c \) in Eq. 2.18 and Eq. 2.19. As seen in Eq. 2.19, the value of \( \frac{f_{c2\text{max}}}{f'_c} \) is reduced by increasing the principal tensile strain \( \varepsilon_1 \).

The concept of tension stiffening is also presented in the MCFT model. The authors suggested that the principal tensile stress prior to cracking is linearly related to the principal tensile strain by the relationship:

\[ f_{c1} = E_c \cdot \varepsilon_1 \quad \text{for} \quad \varepsilon_1 \leq \varepsilon_{cr} \quad \text{Eq. 2.20} \]

where \( E_c \) is the elastic modulus of the concrete and \( \varepsilon_{cr} \) is the cracking concrete strain. For simplicity, \( E_c \) can be taken as \( \frac{2f'_c}{\varepsilon'_c} \). The post-cracking relationship suggested is:

\[ f_{c1} = \frac{f_{cr}}{1 + \sqrt{200\varepsilon_1}} \quad \text{for} \quad \varepsilon_1 > \varepsilon_{cr} \quad \text{Eq. 2.21} \]

For typical concrete design mixes, diagonal cracking will occur at the interface between the aggregate and the cement paste resulting in the transfer of shear stresses through the interlocking of aggregate particles. The contributing factors involved in the aggregate
interlock mechanism, represented in Figure 2.15, include the average crack width over the crack surface \( w \), the average shear stress across the crack \( \nu_{ci} \) and the average compressive stress normal to the crack surface \( f_{ci} \).

![Figure 2.15: Aggregate interlock shear transfer mechanism (Collins & Vecchio, 1986)](image)

The relationship between these factors was derived from the experimental work conducted by Walraven on aggregate interlock (Walraven, 1981). Walraven’s work led to the formulation of the following equations relating \( w \) to the principal compressive strain and the crack spacing \( s_{m\theta} \):

\[
W = \varepsilon_1 \cdot s_{m\theta} \tag{Eq. 2.22}
\]

\[
s_{m\theta} = \frac{1}{\sin \theta} \left( \frac{\cos \theta}{s_{mx}} + \frac{1}{s_{my}} \right) \tag{Eq. 2.23}
\]
In Eq. 2.23, $s_{mx}$ and $s_{my}$ represent the crack spacing in the longitudinal and vertical directions, respectively. For uniform tensile strain, these parameters are determined using the following expressions:

$$s_{mx} = 2\left(c_x + \frac{s_x}{10}\right) + 0.25k_1 \frac{d_{bx}}{\rho_x}$$ \hspace{1cm} \text{Eq. 2.24}

$$s_{my} = 2\left(c_y + \frac{s_y}{10}\right) + 0.25k_1 \frac{d_{by}}{\rho_y}$$ \hspace{1cm} \text{Eq. 2.25}

where:

- $c_x$ is the distance between midsection and the longitudinal reinforcement, mm
- $c_y$ is the distance between midsection and the transverse reinforcement, mm
- $s_x$ is the spacing of longitudinal reinforcement, mm
- $s_y$ is the spacing of transverse reinforcement, mm
- $k_1$ is a coefficient for the bond characteristic of reinforcing bars taken as 0.4 for deformed bars and 0.8 for plain bars
- $d_{bx}$ is the bar diameter of longitudinal reinforcement, mm
- $d_{by}$ is the bar diameter of transverse reinforcement, mm

### 2.6.3 Simplified Modified Compression Field Theory

In the years following the publication of the MCFT, researchers performed multiple tests on similar shear specimens in order to create a model that would enable designers to have a simpler approach to the MCFT procedure. In 2006 Bentz et al. published the Simplified Modified Compression Field Theory that proved to be capable of predicting the shear strength of a wide range of reinforced concrete elements with nearly the same accuracy.
as the MCFT (Bentz et al., 2006). Later this procedure was adopted by the Canadian Standards Association and used as the basis for the shear provisions in the CSA design code, discussed in Section 2.7.3.

2.6.4 Strut and Tie Model

One of the main advantages of designing reinforced concrete beams using the truss model is that the flow of stresses can be easily visualized. Compressive forces are carried by the diagonal concrete struts, while tensile stresses are carried by the reinforcement. As previously discussed, the shear design of reinforced concrete structures depends upon the location of the section of interest, mainly within B-Regions where beam theory is applicable and D-Regions where it is not. Concurrently, it is inappropriate to assume that plane sections remain plane and that shear stresses are uniform over the cross section of D-Regions. Sectional design is inherently dangerous and depends greatly on several variables including the slenderness ratio $a/d$, the presence of openings, and changes in the cross section. Danger also presents itself in the ability of the design engineer to accurately predict the flow of stresses throughout the section, allowing for the possibility of overlooking shear-critical regions such as frame corners and corbels. It is this reason that necessitated a unified approach to the shear design of D-Regions, which was presented in the Strut and Tie Model (STM).

The STM is a conservative upper-bound design procedure in which the design capacity of a structure is limited to the capacity of a single load path, which is less than the actual capacity due to the likelihood that several load paths exist (Rogowsky & MacGregor,
Components of the STM were adapted from the truss model including struts, ties, nodes, and nodal zones.

Struts are internal concrete compression members that carry uniaxial compressive stress, generally idealized as prismatic or uniformly tapered members. However, when strut stresses are allowed to spread into adjacent areas of confining concrete, the strut may be idealized as a bottle-shaped strut. Bottle-shaped struts tend to be narrower at the beginning and end of the strut with a bulge in the middle where this spread of lateral stresses is allowed to occur. In general, strut geometry is determined by the geometry of the overall structure, supports, and externally applied loads.

Ties are uniaxial tension members that consist of prestressed or non-prestressed steel reinforcement. They carry all of the tensile forces throughout the model and are permitted to reach stress levels up to the yield stress \( f_y \) for non-prestressed ties and \( f_y + \Delta f_p \) for prestressed, where \( \Delta f_p \) is equal to 60 ksi. This value is provided based upon the assumption that 60 ksi is a reasonable approximation to the change in stress in the prestressed reinforcement as a member is loaded to failure. Reinforcement typically consists of several bars where the centroid of the steel distribution is concentric with the centroid of the truss tension member. Anchorage is provided by welding the ends to plates or through end hooks with sufficient bond development length.

The axial forces in struts and ties all intersect at locations known as nodes. The area surrounding a node is referred to as the nodal zone, where all of the members are connected through static equilibrium. Nodal classification is identified by the forces acting on the nodal
zone which are composed of four possible combinations: C-C-C, C-C-T, C-T-T, and T-T-T, where “C” represents uniaxial compression and “T” represents uniaxial tension. The classification of the four nodal types is illustrated in Figure 2.16, while Figure 2.17 shows a description of the STM.

![Diagram of nodal types](image)

*(a) C-C-C Node  (b) C-C-T Node  (c) C-T-T Node  (d) T-T-T Node*

*Figure 2.16: Classification of nodes (ACI 318-08)*
The model illustrated in Figure 2.17 consists of two concrete compressive struts, longitudinal reinforcement provided as the tension tie, and the joint nodes and surrounding nodal zones. Unlike traditional shear design procedures which ignore D-regions, Section 11.1.1 of the ACI 318-08 code allows the design of such regions using strut-and-tie models according to the guidelines in Appendix A. Design of concrete structures using the STM was able to be codified under the consideration of several important factors (ACI 445-1, 2002):

a) the geometric layout of strut-and-tie models,

b) what effective concrete strengths and $\phi$ factors should be used,

c) the shape and strength of the struts,

d) the arrangement and strength of the nodal zones

e) the layout, strength, and anchorage of the ties, and

f) the detailing requirements.
Design is performed by first analyzing the flow of stresses throughout the region, computing the factored loads and reactions, and then creating the layout for the strut-and-tie model where the locations of tension ties are determined through static nodal equilibrium. The amount of steel reinforcement to satisfy the tension forces is then computed, along with the effective strut widths and the shapes of the nodal zones.

2.7 Design Codes

There are many code provisions used throughout the world for the shear design of reinforced concrete structures. In the United States the AASHTO (American Association of State and Highway Transportation Officials) LRFD code is most commonly used for the design of bridges while the Building Code Requirements for Structural Concrete provided by the American Concrete Institute (ACI 318-08) is the most common design code for all other reinforced concrete structures. The shear design model used by the AASHTO-LRFD code is based on the Modified Compression Field Theory, while ACI 318-08 is purely based on empirical equations and a 45° truss model. The following chapter discusses several of the most widely used national codes of practice, including the ACI 318-08 provisions, AASHTO LRFD Bridge Design Specifications (2007), Eurocode EC2, and CSA A23.3-04 (Canadian Standards Association: Design of Concrete Structures, 2004).

2.7.1 American Concrete Institute (ACI 318-08)

According to ACI, the shear strength of a reinforced concrete member is based on the average shear stress on the full effective area of the web cross section. For members without any shear reinforcement the shear strength is carried completely by the concrete cross section.
and longitudinal steel within the web, taken as the concrete component to the shear strength $V_c$. When the design shear force $V_u$ is equal to or exceeds $\phi V_c/2$, where $\phi$ is the strength reduction factor, then steel shear reinforcement is required. This reinforcement is typically provided by stirrups spaced at a distance “$s$” along the length of the member. The full shear strength of the section is then controlled partially by the concrete contribution $V_c$ and the steel contribution $V_s$. As previously stated, the equation for $V_s$ in ACI 318-08 is based on a 45° truss model. This means that shear cracks passing across the legs of shear reinforcement are assumed to be inclined at an angle of 45° from the horizontal, creating a more conservative design approach as previously discussed in section 2.6.1. The equations provided by ACI 318-08 for the shear design of normal-weight, non-prestressed reinforced concrete are given in Chapter 11 of the ACI-318-08 code requirements as:

\[
\phi V_n \geq V_u \quad \text{Eq. 2.26}
\]

\[
V_n = V_c + V_s \quad \text{Eq. 2.27}
\]

\[
V_c = \left(1.9\lambda\sqrt{f' c} + 2500\rho_w \frac{V_u d}{M_u}\right)b_w d \leq 3.5\sqrt{f' c} b_w d \quad \text{Eq. 2.28}
\]

Simplified Version: $V_c = 2\lambda\sqrt{f' c} b_w d \quad \text{Eq. 2.29}$

\[
A_{v,min} = 0.75\sqrt{f' c} \frac{b_w s}{f_y t} \geq \frac{50b_w s}{f_y t} \quad \text{Eq. 2.30}
\]

\[
V_s = \frac{A_v f_y t d}{s} \quad \text{Eq. 2.31}
\]
where:

$V_u$ is the factored shear force on the section, lb

$V_n$ is the nominal shear strength of the section, lb

$V_c$ is the nominal shear strength provided by concrete, lb

$\Phi$ is the strength reduction factor taken for shear as 0.75 as provided in section 9.3.2.3 of ACI 318-08.

$V_s$ is the nominal shear strength provided by shear reinforcement, lb

$\lambda$ is the lightweight concrete modification factor taken as 1.0 for normal-weight concrete as provided in section 8.6.1 of ACI 318-08.

$f'_c$ is the specified concrete compressive strength, psi

$\rho_w$ is the ratio of $A_s/b_w d$

$A_s$ is the area of non-prestressed longitudinal tension reinforcement, in$^2$

$b_w$ is the web width, in

$d$ is the effective depth measured from the extreme compression fiber to the centroid of the longitudinal tension reinforcement, in

$M_u$ is the applied ultimate moment on the section, lb-in

$s$ is the center-to-center spacing of the transverse reinforcement, in

$A_v$ is the area of the shear reinforcement, in$^2$

$f_{yt}$ is the specified yield strength of the transverse reinforcement, psi
2.7.2  **AASHTO LRFD Bridge Design Specification (2007)**

The AASHTO-LRFD Bridge Design Specifications are based on the fundamentals presented in the Modified Compression Field Theory rather than empirical equations, unlike the ACI 318-08 code provisions. A sectional design model may be used for the shear design of prestressed and non-prestressed concrete members in accordance with the provisions of Article 5.8.1 in the specifications. The nominal shear resistance, \( V_n \), shall be determined as the lesser of:

\[
V_n = V_c + V_s + V_p \tag{Eq. 2.32}
\]

\[
V_n = 0.25 f'_c b_v d_v + V_p \tag{Eq. 2.33}
\]

in which:

\[
V_c = 0.0316 \beta \sqrt{f'_c b_v d_v}, \text{ if the procedures of Articles 5.8.3.4.1 or 5.8.3.4.2 are used} \tag{Eq. 2.34}
\]

\[
V_c = \text{the lesser of } V_{ci} \text{ and } V_{cw}, \text{ if the procedures of Article 5.8.3.4.3 are used} \tag{Eq. 2.35}
\]

\[
V_{ci} = 0.02 \sqrt{f'_c b_v d_v} + V_d + \frac{V_l M_{cre}}{M_{max}} \geq 0.06 \sqrt{f'_c b_v d_v} \tag{Eq. 2.36}
\]

\[
V_{cw} = \left(0.06 \sqrt{f'_c} + 0.30 f_{pc}\right) b_v d_v + V_p \tag{Eq. 2.37}
\]

\[
V_s = \frac{A_v f_y d_v (\cot \theta + \cot \alpha) \sin \alpha}{s} \tag{Eq. 2.38}
\]
where:

\[ V_p \] is the vertical component of the effective prestressing force, kips

\[ d_v \] is the effective shear depth as determined in Article 5.8.2.9 as the greater of \(0.9d\) or \(0.72h\), in

\[ b_v \] is the effective web width taken as the minimum web width within the depth \(d_v\), in

\[ \beta \] is the factor indicating ability of diagonally cracked concrete to transmit tension

\[ V_d \] is the shear force at the section of interest due to unfactored dead load, kips

\[ V_i \] is the factored shear force at the section of interest due to externally applied loads occurring simultaneously with \(M_{max}\), kips

\[ M_{max} \] is the maximum factored moment at the section of interest due to externally applied loads, kip-in

\[ M_{cr} \] is the moment causing flexural cracking at the section of interest due to externally applied loads, kip-in

\[ V_{cw} \] is the nominal shear resistance provided by concrete when inclined cracking results from excessive principal tensions in the web, kips

\[ f_{pc} \] is the compressive stress in the concrete (after allowance for all prestress losses) at the centroid of the cross section resisting externally applied
loads or at the junction of the web and flange when the centroid lies within the flange, ksi

\( f'_c \) is the specified concrete compressive strength, ksi

\( \theta \) is the angle of inclination of diagonal compressive stresses, degrees

\( \alpha \) is the angle of inclination of transverse reinforcement to the longitudinal axis, degrees

\( f_y \) is the specified yield strength of the transverse reinforcement, ksi

For sections containing at least the minimum amount of transverse reinforcement specified in Article 5.8.2.5 the values of \( \beta \) and \( \theta \) can be obtained using Table 2.2. The design engineer will first choose the appropriate row corresponding to the calculated shear design stress ratio \( \frac{V}{f'_c} = \frac{V_u}{b_v d_v f'_c} \) at the section of interest, and then select the column corresponding to the longitudinal strain \( \varepsilon_c \). For Table 2.2, the longitudinal strain may be determined as:

\[
\varepsilon_x = \frac{|M_u| + 0.5N_u + 0.5|V_u - V_p| \cot \theta - A_p f_y}{2(E_s A_s + E_p A_{ps})}\quad \text{Eq. 2.39}
\]

For sections containing less than the minimum amount of transverse reinforcement specified in Article 5.8.2.5, the values of \( \beta \) and \( \theta \) can be obtained using Table 2.3. The design engineer should select the appropriate row corresponding to the equivalent spacing parameter \( s_{xe} \) at the section of interest, and then select the column corresponding to the longitudinal strain \( \varepsilon_c \). For Table 2.3, the longitudinal strain may be determined as:
\[
\varepsilon_x = \frac{|M_u| + 0.5N_u + 0.5|V_u - V_p| \cot \theta - A_{ps}f_{po}}{(E_sA_s + E_pA_{ps})} \quad \text{Eq. 2.40}
\]

and:

\[
s_{xe} = s_x \frac{1.38}{a_g + 0.63} \leq 80 \text{ in.} \quad \text{Eq. 2.41}
\]

If the strain value calculated from either Eq. 2.40 or Eq. 2.41 is negative, then \( \varepsilon_s \) shall be taken as:

\[
\varepsilon_x = \frac{|M_u| + 0.5N_u + 0.5|V_u - V_p| \cot \theta - A_{ps}f_{po}}{2(E_cA_c + E_sA_s + E_pA_{ps})} \quad \text{Eq. 2.42}
\]

Where:

- \( A_c \) is the area of concrete on the flexural tension side of the member, in\(^2\)
- \( A_{ps} \) is the area of prestressing steel on the flexural tension side of the member, in\(^2\)
- \( A_s \) is the area of non-prestressed steel on the flexural tension side of the member at the section under consideration, in\(^2\)
- \( f_{po} \) is the modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete, ksi
- \( N_u \) is the factored axial force, taken as positive in tensile and negative if compressive, kips
$M_u$ is the factored moment, not to be taken less than $V_u d_v$, kip-in

$V_u$ is the factored shear, kips

$s_x$ is the crack spacing parameter taken as the lesser of either $d_v$ or the maximum distance between layers of longitudinal crack control reinforcement, where the area of the reinforcement in each layer is not less than $0.003 b_v s_x$, in

$a_g$ is the maximum aggregate size, in
### Table 2.2: Members with at least minimum shear reinforcement (NCHRP, 2005)

<table>
<thead>
<tr>
<th>$y^{*}/f_{c}$</th>
<th>Longitudinal Strain, $\varepsilon_x \times 1000$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\leq -0.20$</td>
</tr>
<tr>
<td>$\leq 0.075$</td>
<td>$\theta$ 22.3° 20.4° 21.0° 21.8° 24.3° 26.6° 30.5° 33.7° 36.4° 40.8° 43.9°</td>
</tr>
<tr>
<td>$\leq 0.100$</td>
<td>$\theta$ 18.1° 20.4° 21.4° 22.5° 24.9° 27.1° 30.8° 34.0° 36.7° 40.8° 43.1°</td>
</tr>
<tr>
<td>$\leq 0.125$</td>
<td>$\theta$ 19.9° 21.9° 22.8° 23.7° 25.9° 27.9° 31.4° 34.4° 37.0° 41.0° 43.2°</td>
</tr>
<tr>
<td>$\leq 0.150$</td>
<td>$\theta$ 21.6° 23.8° 24.2° 25.0° 26.9° 28.8° 32.1° 34.9° 37.3° 40.5° 42.8°</td>
</tr>
<tr>
<td>$\leq 0.175$</td>
<td>$\theta$ 23.2° 24.7° 25.5° 26.2° 28.0° 29.7° 32.7° 35.2° 36.8° 39.7° 42.2°</td>
</tr>
<tr>
<td>$\leq 0.200$</td>
<td>$\theta$ 24.7° 26.1° 26.7° 27.4° 29.0° 30.6° 32.8° 34.5° 36.1° 39.2° 41.7°</td>
</tr>
<tr>
<td>$\leq 0.225$</td>
<td>$\theta$ 26.1° 27.3° 27.9° 28.5° 30.0° 30.8° 32.3° 34.0° 35.7° 38.8° 41.4°</td>
</tr>
<tr>
<td>$\leq 0.250$</td>
<td>$\theta$ 27.5° 28.6° 29.1° 29.7° 30.6° 31.3° 32.8° 34.3° 35.8° 38.6° 41.2°</td>
</tr>
</tbody>
</table>

### Table 2.3: Members with less than minimum shear reinforcement (NCHRP, 2005)

<table>
<thead>
<tr>
<th>$\varepsilon_x$ (in)</th>
<th>Longitudinal Strain, $\varepsilon_x \times 1000$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\leq -0.20$</td>
</tr>
<tr>
<td>$\leq 5$</td>
<td>$\theta$ 25.4° 25.5° 25.9° 26.4° 27.7° 28.9° 30.9° 32.4° 33.7° 35.6° 37.2°</td>
</tr>
<tr>
<td>$\leq 10$</td>
<td>$\theta$ 27.6° 27.6° 28.3° 29.3° 31.6° 33.5° 35.3° 38.4° 40.1° 42.7° 44.7°</td>
</tr>
<tr>
<td>$\leq 15$</td>
<td>$\theta$ 29.5° 29.5° 29.7° 31.1° 34.1° 36.5° 39.9° 42.4° 44.4° 47.4° 49.7°</td>
</tr>
<tr>
<td>$\leq 20$</td>
<td>$\theta$ 31.2° 31.2° 31.2° 32.3° 36.0° 38.8° 42.7° 45.5° 47.6° 50.9° 53.4°</td>
</tr>
<tr>
<td>$\leq 30$</td>
<td>$\theta$ 34.1° 34.1° 34.1° 34.2° 38.9° 42.3° 46.9° 50.1° 52.6° 56.2° 59.0°</td>
</tr>
<tr>
<td>$\leq 40$</td>
<td>$\theta$ 36.6° 36.6° 36.6° 36.6° 41.1° 45.0° 50.2° 53.7° 56.3° 60.2° 63.0°</td>
</tr>
<tr>
<td>$\leq 60$</td>
<td>$\theta$ 40.8° 40.8° 40.8° 40.8° 44.5° 49.2° 55.1° 58.9° 61.8° 65.8° 68.6°</td>
</tr>
<tr>
<td>$\leq 80$</td>
<td>$\theta$ 44.3° 44.3° 44.3° 44.3° 47.1° 52.3° 58.7° 62.8° 65.7° 69.7° 72.4°</td>
</tr>
</tbody>
</table>
2.7.3 Canadian Standards Association CSA A23.3-04

Similar to the AASHTO LRFD specifications, the shear design equations adopted by the Canadian Standards Association (CSA 2004) are derived from the Simplified Modified Compression Field Theory. Unlike AASHTO LRFD, however, the CSA provisions provide a much easier and less cumbersome design procedure and have eliminated the use of tables to calculate variables such as $\beta$ and $\theta$. The equations used for the shear design at mid-depth of a section are presented below in metric units:

\[ V_r = V_c + V_s + V_p \]  \hspace{1cm} \text{Eq. 2.43}

\[ V_{r,\text{max}} = 0.25 f'_c b_w d_v + V_p \]  \hspace{1cm} \text{Eq. 2.44}

\[ V_c = \phi_c \lambda \beta \sqrt{f'_c b_w d_v} \]  \hspace{1cm} \text{Eq. 2.45}

\[ V_s = \frac{\phi_s A_v f_y d_v \cot \theta}{s} \]  \hspace{1cm} \text{Eq. 2.46}

\[ \beta = \frac{0.40}{(1 + 1500 \varepsilon_x)} \cdot \frac{1300}{(1000 + s_{ze})} \]  \hspace{1cm} \text{Eq. 2.47}

\[ \theta = 29 + 7000 \varepsilon_x \]  \hspace{1cm} \text{Eq. 2.48}

\[ s_{ze} = \frac{35 s_x}{15 + a_g} \]  \hspace{1cm} \text{Eq. 2.49}

\[ A_v = 0.06 \sqrt{f'_c} \frac{b_w s}{f_y} \]  \hspace{1cm} \text{Eq. 2.50}

\[ \frac{M_f}{A_v} + V_f - V_p + 0.5N_f - A_p f_{po} \]  \hspace{1cm} \text{Eq. 2.51}

\[ \varepsilon_x = \frac{\frac{M_f}{A_v} + V_f - V_p + 0.5N_f - A_p f_{po}}{2(E_s A_s + E_p A_p)} \]
where:

\[ \lambda \] is a reduction factor that accounts for low-density concrete

\[ s_{ze} \] is the equivalent crack spacing parameter that allows for aggregate size, mm

\[ s_z \] is a crack spacing parameter taken as \( d_v \) or as the maximum distance between layers of distributed longitudinal reinforcement, mm

2.7.4 The Eurocode 2

The design of a beam with shear reinforcement using the Eurocode is based on the variable angle truss model where the concrete strut angle can be varied within limits to provide the required accuracy. As this code is used throughout Europe, each country has its own National Application (NA) document which dictates what factors should be used. The design is controlled by the minimum of the capacity of the concrete or that of the steel, unlike the other codes where the capacity is controlled by the sum of the concrete and steel contributions. For members without transverse reinforcement, Eurocode specifies the following equation in metric units:

\[ V_c = \left[ c_{Rd,c} K (100 \rho_w f'_{c_r})^{1/3} \right] b_w d \quad \text{Eq. 2.52} \]

with:

\[ V_{c,\text{min}} = (v_{\text{min}} + k_1 \sigma_{cp}) b_w d \quad \text{Eq. 2.53} \]

where:
$C_{Rd,c}$ is the factor of safety of the concrete, equal to $0.18/Y_c$

$K$ is the factor which takes into account the size effect equal to

$$1 + \sqrt{200/d} \leq 2.0$$

$v_{min}$ is the minimum shear stress equal to $v_{min} = 0.035K^{3/2}\sqrt{f'^c}$

$k_1$ is a constant equal to 0.15 in the British Annex of the Eurocode

$\sigma_{cp}$ is the axial stress on the cross section equal to $N_{ed}/A_c$ where $N_{ed}$ is the axial force due to loading or prestressing

For beams with transverse reinforcement, the Eurocode recommends the use of the minimum of the following equations for the evaluation of the beam shear resistance:

$$V_s = \frac{A_v f_{y_t} d_v}{s} \cot \theta \quad \text{Eq. 2.54}$$

$$V_s = \frac{\alpha_{cw} b_w d_v v_1 f'^c}{(\cot \theta - \tan \theta)} \quad \text{Eq. 2.55}$$

where:

$\alpha_{cw}$ is a coefficient taking into account the state of stress in the compression chord

$v_1$ is a strength reduction factor for concrete cracked in shear equal to 0.6 for concrete with $f'^c \leq 60$ MPa
The Eurocode specifies that the shear resistance of a beam with transverse reinforcement is the lesser of Eq. 2.54 and Eq. 2.55 representing the steel contribution to the shear resistance, rather than the sum of this value and the concrete contribution to the shear capacity. Neglecting $V_c$ in this way increases the conservatism of the Eurocode with respect to other codes that do not.
3.0 EXPERIMENTAL PROGRAM

3.1 Introduction

This chapter describes the experimental program undertaken at the Constructed Facilities Laboratory (CFL) located on North Carolina State University’s Centennial Campus in Raleigh, North Carolina. The program was designed to examine the shear behavior of large scale LSP reinforced concrete members. The research varied the unit weight and compressive strength of normalweight concrete by adding different amounts of LSP to the mix design, as well as the shear span-to-depth ratio, $a/d$, and the transverse reinforcement ratio, $\rho_v$. The LSP used in this experimental program is commercially known as Elemix Type XE concrete additive.

3.2 Test Specimens

The shear behavior of reinforced concrete members made with LSP was evaluated using nine beams, each tested twice for a total of eighteen tested specimens. All of the beams had nominal cross sectional dimensions of 12”x18” and a total length of 16’. The specimens were divided into two main groups with two targeted unit weights of 120 lb/ft$^3$ and 130 lb/ft$^3$. Two targeted concrete compressive strengths were also used, with 2,500 psi corresponding to a unit weight of 120 lb/ft$^3$, and 4,000 psi corresponding to a unit weight of 130 lb/ft$^3$. Furthermore, the specimens with the target unit weight of 130 lb/ft$^3$ were divided into two subgroups with different shear span-to-depth ratios. A test matrix for the experimental program is presented in Figure 3.1.
The beam test specimens were identified using four parameters: the first letter “S” indicates that the specimen was tested in shear, the second identifies the targeted unit weight in pounds per cubic feet “120” or “130”, the third parameter defines the transverse reinforcement ratio “0%, 0.25%, and 0.5%” for the beams without transverse reinforcement, with minimum stirrups, and maximum stirrups, respectively. A number “3” or “1.5” following the transverse reinforcement ratio refers to the shear span-to-depth ratio used for each set of beams. A letter “R” at the end designates the replicated specimens since each beam was tested twice.

The transverse reinforcement for the test specimens was provided in the form of closed stirrups which were designed according to the ACI 318-08 provisions for shear. Some of the beams were designed without transverse reinforcement to evaluate the concrete
contribution to shear strength, $V_c$. Others were designed with minimum and maximum allowable transverse reinforcement as defined by ACI 318-08 to demonstrate the range of strength increase using transverse reinforcement. The minimum amount of transverse reinforcement was determined based on the maximum stirrup spacing as determined by the ACI Code 11.4.5.1, which is equal to $d/2$. This value was compared to $s_{\text{max}}$ which corresponds to the minimum area of shear reinforcement as determined by 11.4.6.3 of the ACI Code. The controlling maximum spacing was determined to be 7.2 inches but a value of 7.0 inches was chosen for constructability of the test specimens. The maximum amount of transverse reinforcement was controlled by the allowable spacing provision in 11.4.5.3 which reduced the maximum stirrup spacing by one half. The resulting value of 3.5 inches was taken as the minimum stirrup spacing which complied with 11.4.7.9, which requires that the steel contribution, $V_s$, not exceed $8\sqrt{f'_c} b_w d$.

As previously mentioned in Section 2.4, concrete made with the LSP additive using normalweight aggregates is classified as normalweight concrete. Therefore, in this study, the parameter $\lambda$ normally associated with lightweight concrete is not used in the design of the beams used in the experimental program.

The flexural reinforcement for the beams was designed in order to ensure that each test specimen failed in shear. For beams without stirrups, the shear load is carried completely by the concrete component, $V_c$, and conversely for beams with stirrups, the load is carried by the summation of both concrete and steel components, $V_c + V_s$. 
In order to obtain a shear-controlled failure, the flexural capacity of a reinforced concrete beam must be greater than the demand due to the applied load, while at the same time ensuring that the applied shear force is greater than the ultimate shear load carrying capacity of the beam. The flexural reinforcement was kept constant for all of the beams in order to prevent variability of dowel action and the ACI stress block was used to calculate the nominal moment capacity and design the reinforcement necessary to accommodate the moment demand. Tension reinforcement was provided by one layer of 4 No. 9 bars and one layer of 2 No. 9 bars with a spacing of one No. 9 bar diameter above the bottom layer. Compression reinforcement was provided by one layer of 4 No. 10 bars. The protection of reinforcement provided by concrete cover was determined to be 1.5” from the exposed surface to the stirrups, as per section 7.7.1 of the ACI Code. The typical cross section for all test specimens is illustrated in Figure 3.2.
Design of the specimens allowed testing of one end of the beam while the other cantilevered end of the beam remained unstressed. After completion of the first test, the beams were rotated and the previously cantilevered end was tested as a second specimen. Therefore within each group, two beams were identical. Two beams were reinforced with the maximum amount of stirrups for 2/3 of the beam length, while the remaining length contained no stirrups. The portion without stirrups was tested first to evaluate the concrete contribution to the shear strength, $V_c$, then the portion with the maximum amount of stirrups was tested to examine the overall shear strength, $V_s$. The third beam in each group was reinforced with the minimum amount of stirrups for the entire length. Figure 3.3 shows the layout and dimensions of the beams in Group 1 and Group 2 tested with a shear span-to-
depth ratio of 3.0. Figure 3.4 shows the layout and dimensions of the beams in Group 3 tested with a shear span-to-depth ratio of 1.5.

Figure 3.3: Group 1 and Group 2 beams (a/d = 3.0)
Figure 3.4: Group 3 beams (a/d = 1.5)
All transverse and longitudinal reinforcement was detailed according to the ACI Code requirements in section 7.1. For the longitudinal steel, a standard hook extending twelve bar diameters past the 90° bend in the tension steel was used at each end in order to achieve sufficient bond, while the compression steel was kept straight throughout the length of each beam. The transverse steel consisted of #3 closed stirrups with six bar diameters extending past the 90° bend. The final design matrix for all of the test specimens is shown in Table 3.1.
Table 3.1: Design details of the test specimens

<table>
<thead>
<tr>
<th>ID</th>
<th>$b \times h$</th>
<th>cover</th>
<th>$d$</th>
<th>$L$</th>
<th>$a / d$</th>
<th>$a$</th>
<th>Specified $f'_c$</th>
<th>Tension bars</th>
<th>Compression bars</th>
<th>Stirrups</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>in</td>
<td>in</td>
<td>in</td>
<td>ft</td>
<td>in</td>
<td>psi</td>
<td>No. of bars</td>
<td>Bar Size #</td>
<td>No. of bars</td>
<td>Bar Size #</td>
</tr>
<tr>
<td>S-120-0%-3 (R)</td>
<td>12 x 18</td>
<td>1.5</td>
<td>14.4</td>
<td>16</td>
<td>3</td>
<td>43</td>
<td>2500</td>
<td>6</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>S-120-0.25%-3 (R)</td>
<td>12 x 18</td>
<td>1.5</td>
<td>14.4</td>
<td>16</td>
<td>3</td>
<td>43</td>
<td>4000</td>
<td>6</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>S-130-0%-3 (R)</td>
<td>12 x 18</td>
<td>1.5</td>
<td>14.4</td>
<td>16</td>
<td>3</td>
<td>43</td>
<td>2500</td>
<td>6</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>S-130-0.25%-1.5 (R)</td>
<td>12 x 18</td>
<td>1.5</td>
<td>14.4</td>
<td>9</td>
<td>1.5</td>
<td>22</td>
<td>4000</td>
<td>6</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>S-130-0.5%-1.5 (R)</td>
<td>12 x 18</td>
<td>1.5</td>
<td>14.4</td>
<td>9</td>
<td>1.5</td>
<td>22</td>
<td>4000</td>
<td>6</td>
<td>9</td>
<td>4</td>
</tr>
</tbody>
</table>
3.3 **Fabrication of the Test Specimens**

The test specimens were fabricated at the CFL using steel cages that were first tied by hand, as shown in Figure 3.5. The figure shows a portion of the beam without transverse reinforcement which will be tested to evaluate the concrete contribution to the shear strength; the remainder of the beam has either the maximum or minimum amount of stirrups allowed by the ACI 318-08 Building Code and tested to evaluate the strength increase due to transverse reinforcement.

![Finished beam cages](image-url)
After construction of the cages was completed, they were placed into U-shaped steel-ply formwork located outside of the laboratory on a level casting bed. Before insertion of the steel cages, the formwork was sprayed with a chemical agent to facilitate release of the cured concrete beams. Figure 3.6 shows completed steel cages inside of the formwork.

Figure 3.6: Steel cages in finished formwork
The concrete was supplied by a local ready-mixed company and the LSP was added directly into the concrete truck. After allowing sufficient time for the LSP additive to disperse evenly throughout the mix, the concrete was then poured directly into the formwork on the casting bed, as shown in Figure 3.7. The concrete was consolidated using an electric vibrator and the top surface of each beam was finished using hand-held finishing trowels. Once the concrete had set, the beams were covered in damp burlap and covered with plastic sheets and a plastic tarp in order to minimize the moisture lost during curing. The beams remained protected for a minimum of seven days when the formwork could be stripped and they could be moved. In order to test the compressive strength of the concrete, 4” x 8” cylinders were cast and placed beside the finished beams in order to obtain actual material properties.

Figure 3.7: Casting LSP concrete
3.4 Material Properties

In order to correctly analyze and predict the behavior of the test specimens, material properties were obtained for both the steel and the concrete used in the experimental program.

3.4.1 Concrete Properties

The concrete used for fabricating the beams was provided by a local ready-mixed concrete company and consisted of Lehigh Type I cement, Class F fly ash, ASTM C33 natural sand, #78 granite, Elemix® concrete additive – Type XE, and standard HRWR. From the experimental program, it can be seen that there were two batches of concrete cast with unit weights of 120 lb/ft³ and 130 lb/ft³ corresponding to target 28 day compressive strengths of 2500 psi and 4000 psi, respectively. A sample mix design for each batch of concrete can be seen in Error! Reference source not found. and the actual batch weights for each cast are summarized in Table 3.3.

| Table 3.2: Sample of concrete mixes for test specimens |
|---------------------------------------------|----------------|-------------|-------------|-------------|--------------|----------------|----------------|
| Target Unit Weight | Cement | Class F Fly Ash | Natural Sand | #78 Granite | Elemix Concrete Additive Type XE | W/CM | Water Reducer | HRWR |
| lb/ft³ | lb/cy | lb/cy | lb/cy | lb/cy | oz/cwt | oz/cwt |
| By Weight 120 | 752 | 188 | 1380 | 722 | 9 | 0.4 | 4 | 3 |
| By Weight 130 | 677 | 200 | 1675 | 914 | 4 | 0.4 | 4 | 5 |
Table 3.3: Sample of concrete batch weights

<table>
<thead>
<tr>
<th>Cast</th>
<th>Target Unit Weight</th>
<th>Cement</th>
<th>Class F Fly Ash</th>
<th>Natural Sand (moisture content)</th>
<th>#78 Granite (moisture content)</th>
<th>Elemix Concrete Additive-Type XE</th>
<th>Added Water</th>
<th>W/CM</th>
<th>Water Reducer</th>
<th>HRWR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lb/ft³</td>
<td>lb/cy</td>
<td>lb/cy</td>
<td>lb/cy</td>
<td>lb/cy</td>
<td>lb/cy</td>
<td>lb/cy</td>
<td></td>
<td>oz/cwt</td>
<td>oz/cwt</td>
</tr>
<tr>
<td>1</td>
<td>120</td>
<td>748</td>
<td>188</td>
<td>1330 (9.0%)</td>
<td>705 (1.0%)</td>
<td>9</td>
<td>209</td>
<td>0.36</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>130</td>
<td>675</td>
<td>197</td>
<td>1723 (13.3%)</td>
<td>892 (0.4%)</td>
<td>4</td>
<td>161</td>
<td>0.45</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

For each cast, 4” x 8” cylinders were made in accordance with ASTM C31 and placed beside the specimens to reflect the specimens’ curing conditions. Three cylinders were tested at three, seven, fourteen, and twenty-eight days in accordance with ASTM C39 to obtain the compressive strength for each batch. Three additional cylinders were also cast for each beam in order to obtain the concrete compressive strength for the test specimens at the time of testing.

For cast 1, the measured concrete compressive strength at 28 days was 2760 psi and was lower than the compressive strength of 3140 psi measured at 14 days. It is believed that this lower compressive strength is attributed to the wet conditions of the cylinders at the time of casting.
of testing. When wet cylinders are tested in compression, the internal pore pressure increases causing premature failure of the cylinders. For this reason, the concrete compressive strength at 28 days was determined using the measured values at 14 days and at the day of testing in accordance with ACI 209R-92 using the following equation:

\[(f'_c)_t = \frac{t}{a + \beta t} (f'_c)_{28}\]  \hspace{1cm} \text{Eq. 3.1}

where:

- \(a\) is a constant depending on the curing method, measured in days;
- \(t\) is the age of the specimen, measured in days;
- \(\beta\) is a constant depending on the curing method;
- \((f'_c)_{28}\) is the concrete compressive strength at 28 days, measured in psi;
- \((f'_c)_t\) is the concrete compressive strength at any age of concrete, measured in psi;

For the concrete used in the experimental program, \(a = 4\) and \(\beta = 0.85\);

At 14 days, \((f'_c)_t = 3140 \text{ psi}\) resulting in \((f'_c)_{28} = 3570 \text{ psi}\);

At the day of testing, on average, \((f'_c)_t = 4120 \text{ psi}\) and \(t = 106 \text{ days}\), resulting in \((f'_c)_{28} = 3670 \text{ psi}\);

The concrete compressive strength at 28 days was taken as the average of these two results:

\[(f'_c)_{28} = \frac{3570 + 3670}{2} = 3620 \text{ psi};\]
### Table 3.4: Concrete compressive strength according to ACI 209R-92

<table>
<thead>
<tr>
<th>Specimens Cast</th>
<th>Measured Unit Weight lb/ft³</th>
<th>Compressive Strength psi</th>
<th>Target</th>
<th>( f'_c )</th>
<th>( f_c ) Day of Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cast 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Target U.W. = 120 lb/ft³</td>
<td>S-120-0%-3 (R)</td>
<td>120.2</td>
<td>2500</td>
<td>3620</td>
<td>4120</td>
</tr>
<tr>
<td></td>
<td>S-120-0.25%-3 (R)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-120-0.5%-3 (R)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Cast 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Target U.W. = 130 lb/ft³</td>
<td>S-130-0%-3 (R)</td>
<td>131.1</td>
<td>4000</td>
<td>5930</td>
<td>6890</td>
</tr>
<tr>
<td></td>
<td>S-130-0.25%-3 (R)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-130-0.5%-3 (R)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-130-0%-1.5 (R)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-130-0.25%-1.5 (R)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S-120-0.5%-1.5 (R)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The concrete cylinders were tested using a 500 kips capacity Forney compression testing machine at a load rate ranging between 21,100 lb/min and 31,700 lb/min, as illustrated in Figure 3.8. The cylinders were placed between neoprene caps in order to obtain an even distribution of stresses during testing. The maximum applied load was recorded and used to calculate the ultimate compressive stress of the cylinder based on the average cross-sectional area. The average compressive strength of the concrete for all of the tested specimens is presented in Appendix B.
3.4.2 Steel Properties

Direct tension tests were used to obtain the stress-strain characteristics for both the longitudinal and transverse reinforcing steel. 2’ long tension coupons were tested in accordance with ASTM A370 using an MTS universal tension-compression testing machine with hydraulic grips and a capacity of 220 kips. A Vishay data acquisition system was used to record load and stroke data from the testing machine, as well as the tension strain data that was captured using an MTS digital extensometer attached to the bar.

Each coupon sample of No. 3 bars was tested at a load rate of 0.001 inches of grip displacement per second until the applied load necessary to obtain substantial bar elongation had decreased, and then the extensometers were removed to prevent damage due to sudden
rupture of the specimen. Load was applied until fracture occurred shortly after necking. For the No. 9 bars, the load was applied at the same rate; however, the test was stopped after the formation of a definite yield plateau to prevent damage to the testing equipment. Testing of the No. 9 bars is shown in Figure 3.9.

Figure 3.9: Tension test of the #9 bars
A minimum of three coupons each were tested for the No. 9 bars used as the tension reinforcement and the No. 3 bars used as transverse stirrups. Longitudinal reinforcement for all of the test specimens and stirrups for specimens in Group 1 and Group 2 was provided at the same time, while the stirrups used as transverse reinforcement for Group 3 specimens were provided separately. This separation of batches for the No. 3 stirrups required that an additional test be conducted to determine the stress-strain behavior of the separate batch. Figure 3.10 shows the stress-strain behavior of the No. 9 bars, while Figure 3.11 and Figure 3.12 show the stress-strain behavior for the No. 3 stirrups. The No. 10 bars used as compression reinforcement were not tested due to the limited MTS capacity.
Figure 3.10: Stress-strain relationship for #9 reinforcing bars
Figure 3.11: Stress-strain relationship for #3 stirrups (Group 1 & Group 2 beams)
Figure 3.12: Stress-strain relationship for #3 stirrups (Group 3 beams)

3.5 Test Setup

Two test setup configurations were used for the experimental program, one for each of the two shear span-to-depth ratios. The first setup was arranged for Group 1 and Group 2 beams tested with $a/d$ of 3.0 and allowed application of a single point load at a distance of 43” from the left support. The second setup was used for Group 3 beams with $a/d$ of 1.5 and
allowed application of a single point load at a distance of 22” from the left support. The untested portion of each beam was cantilevered over the right support. The beams were rotated after completion of the first test and the unstressed portion of each beam was tested as a second beam. Figure 3.13 and Figure 3.14 show the test setups for beams with shear span-to-depth ratios of 3.0 and 1.5, respectively.

Figure 3.13: Typical test setup for beams with a/d = 3.0
The load was applied using a 440 kips capacity hydraulic actuator supported by a steel frame that was securely anchored to the laboratory strong floor. The load provided by the actuator was transferred to two steel loading plates that measured 1” thick by 8” wide. The beams were simply supported by a steel pin and a 1” thick bearing plate at the left end support and a steel roller and 1” thick bearing plate at the right support. A gypsum cement
paste was used to create a flat surface between the beam and the bearing plate at each support.

3.6 Instrumentation

For each test setup, two 200 kips capacity load cells were placed under the left support to measure the reaction, which represented the maximum shear forces within the shear span under consideration. Two linear potentiometers were used at the top surface of the beams at both supports to detect any possible deformation. Vertical beam deflections were measured by string potentiometers placed directly on the underside of the beam at the location of the actuator, as shown in Figure 3.17 and Figure 3.18. The reported deflection is the relative maximum measured deformation at the point of load application to the measured deformation at the supports.

Stirrup strains were measured using weldable electric resistance strain gages placed on selected stirrups within each beam for specimens with transverse reinforcement. The location of the weldable strain gages was chosen based on the predicted crack path from the point of load application to the support. Each strain gage was carefully applied to the prepared surface of the instrumented stirrup, as specified by the manufacturer, using a specially designed tack welder as shown in Figure 3.15. Figure 3.16 shows the weldable strain gage arrangement for specimen S-130-0.5%-3-R.
Figure 3.15: Weldable strain gage

Figure 3.16: Location of weldable strain gages for specimen S-130-0.5%-3-R
Weldable strain gages were chosen for use in the experimental program due to their excellent durability, performance and bonding capacity. If weldable strain gages were not used in place of conventional electrical resistance strain gages, then the diagonal shear cracks that cross the stirrups at these locations may cause the strain gage to de-bond, resulting in a loss of experimental data.

Crack widths and concrete strains were measured using PI gage rosettes which captured most of the cracking located within the shear span region. The rosettes consisted of three PI gages, each measuring 200 mm in length. The gages were overlapped with one gage placed horizontally, one gage placed vertically, and the other at an angle of 45° from the horizontal axis of the beam. For test specimens in Group 1 and Group 2, three PI gage rosettes were arranged diagonally between the applied load and the left support reaction as shown in Figure 3.17. On the opposite face of the test specimen PI gages were arranged vertically at the location of a specific stirrup located within the shear span region as shown in Figure 3.18. This orientation was used with the intent to measure the stirrup strain at the selected location. However, the results obtained from these PI Gages were deemed insufficiently conclusive and therefore were not used in the remainder of this thesis. For specimens with a shear span-to-depth ratio of 1.5, only two PI gage rosettes were arranged diagonally between the applied load and the left support reaction, with a similar PI gage configuration on the backside as well.
Figure 3.17: Testing instrumentation (front face)
3.7 Loading

Load was applied using a 440 kips capacity, hydraulic MTS actuator and recorded using a Vishay data acquisition system, respectively. For the purposes of this research, displacement-control was used as the loading method by specifying a total displacement over a certain time increment, which was determined to be 0.1 inches of displacement per 150 seconds. The instrumentation readings were scanned using the Vishay data acquisition system.
system at a rate of 1 scan per second. Throughout the duration of a test, time was taken for visual inspection of the beam, measuring crack widths, locating and marking new cracks, and taking photographs. A typical test would take approximately 40 minutes for specimens without stirrups, and 60 minutes for specimens with stirrups.
4.0 EXPERIMENTAL RESULTS AND ANALYSIS

This chapter summarizes the results of the experimental investigation conducted at the CFL, and the comprehensive behavior of the tested specimens based on the measured data. The experimental program consisted of nine reinforced LSP concrete beams, each tested twice for a total of eighteen tested specimens. Also included in the chapter is a discussion of the significance of the test results and comparisons with code approaches to predict the shear load carrying capacity of reinforced concrete beams with LSP additive.

4.1 Cracking behavior

During the early stages of testing, typical flexural cracking behavior was observed for specimens within Group 1 and Group 2. Initiation of the first flexural cracks occurred when the induced stress levels reached the modulus of rupture of the concrete at the location of maximum bending moment directly under the applied load. Flexural cracks propagated upwards toward the compression zone and increased in number with increasing applied load.

Overall, the formation of shear cracks was dependent upon the presence of shear reinforcement and the $a/d$ ratio. For specimens without shear reinforcement in Group 1 and Group 2, the onset of shear cracking appeared as a single diagonal crack that spanned between the support and the point of load application within the shorter shear span. Once this first diagonal shear crack appeared, it propagated through the shear span and increased in width with increasing applied load. Figure 4.1 shows the typical cracking behavior for specimens without transverse reinforcement and $a/d$ ratio of 3.0. For specimens with shear
reinforcement and $a/d$ ratio of 3.0, the first shear cracks appeared as an extension of the flexural shear cracks in the diagonal direction. When the applied load was increased, new diagonal shear cracks formed and could be seen on both faces of the test specimen. Shear cracks increased in width and number with the increasing applied load. At different load intervals, crack widths were measured using hand-held crack width comparators. For Group 1 and Group 2 specimens, in all cases, failure occurred when the member was unable to sustain any more increase of the applied load. Figure 4.2 and Figure 4.3 show the typical shear cracking of specimens with $a/d$ ratio of 3.0 and transverse reinforcement ratios of 0.25% and 0.5%, respectively, showing multiple cracks within the shear span region. A comparison of the cracking behavior for all of the specimens within Group 1 and Group 2 can be found in Appendix A.
Figure 4.1: Typical cracking behavior of Group 1 and Group 2 specimens ($a/d = 3.0, \rho_t = 0\%$)
Figure 4.2: Typical cracking behavior of Group 1 and Group 2 specimens ($a/d=3.0, \rho_t = 0.25\%$)
Figure 4.3: Typical cracking behavior of Group 1 and Group 2 specimens ($a/d = 3.0, \rho_t = 0.5\%$)

The cracking behavior for specimens within Group 3 was much different in comparison to that for specimens within Group 1 and Group 2. Flexural and shear cracks appeared similarly for these specimens, however the crack pattern and the behavior in which cracks propagated was influenced greatly by the enhanced load carrying capacity due to the “arching action” mechanism within the reduced shear span. This was the dominant failure mechanism for Group 3 specimens without transverse reinforcement and the specimens with
the maximum amounts of transverse reinforcement. For this reason, while testing the S-130-0%-1.5 and replicate specimens, the tests were stopped at high load levels in order to prevent damage to the untested portion of the beams. The typical cracking behavior for Group 3 specimens with no transverse reinforcement and $a/d$ ratio of 1.5 can be seen in Figure 4.4. For specimens with the minimum amount of stirrups, failure occurred in the longer shear span ($a/d = 3.0$) of the beam. Even the shorter shear span of the beam continued to carry load as shown in Figure 4.5. When the transverse reinforcement ratio was increased to the maximum allowed value, the beam failed due to arching action within the short shear span with the formation of multiple shear cracks, as shown in Figure 4.6. A comparison of the cracking behavior for all of the specimens within Group 3 can be found in Appendix A.
Figure 4.4: Typical cracking behavior of Group 3 specimens ($a/d = 1.5$, $\rho_t = 0\%$)
Figure 4.5: Typical cracking behavior of Group 3 specimens ($a/d = 1.5$, $\rho_t = 0.25\%$)
Crack widths were monitored continuously using the PI gage rosettes connected to the data acquisition system. They were also manually measured at different load levels using hand-held crack width comparators. As previously mentioned, specimens without transverse reinforcement failed shortly after the initiation of the first diagonal shear crack, while for specimens with transverse reinforcement, this was not the case. The presence of stirrups controlled the increase in width of the shear cracks which led to the formation of multiple

Figure 4.6: Typical cracking behavior of Group 3 specimens (a/d = 1.5, ρt = 0.25%)
diagonal cracks within the shear span region. Figure 4.7 and Figure 4.8 show the average concrete strains measured by the three rosettes during each test of the beams within Group 1 and Group 2. The figures demonstrate the effect of increasing the transverse reinforcement ratio from 0.25% to 0.5%. As more stirrups are introduced within the section, the stresses, and consequently the strains, are relieved through a larger amount of shear reinforcement.

![Figure 4.7: Concrete strain from the rosettes for Group 1 specimens](image-url)

*Figure 4.7: Concrete strain from the rosettes for Group 1 specimens*
Figure 4.8: Concrete strain from the rosettes for Group 2 specimens

Data collected from the PI gage rosettes were used to evaluate the average crack width passing through the rosettes during testing. Eq. 4.1 proposed by Shehata (1999) is derived from Figure 4.9:
\[
\sum w = (\sqrt{2}\Delta_D - \Delta_V - 0.5l_g\varepsilon_{ct}) \sin \theta + (\Delta_V - 0.5l_g\varepsilon_{ct}) \cos \theta
\]

Eq. 4.1

where:

- \( \sum w \) is the crack width based on all of the cracks passing through the PI gage rosette, in.
- \( \Delta_D \) is the measured PI gage reading in the diagonal direction, in.
- \( \Delta_V \) is the measured PI gage reading in the vertical direction, in.
- \( l_g \) is the gage length of the PI gage, 7.87 in. [200 mm]
- \( \varepsilon_{ct} \) is the maximum tensile concrete strain, 0.0001 in/in
- \( \theta \) is the measured crack angle to the horizontal axis of the test beam, degrees
The average crack width is calculated by dividing the summation of the shear crack widths $\sum w$ passing through a rosette by the number of cracks at the corresponding load levels. The crack angles used in the equation were determined based on the angle of crack inclination at failure. The average crack width is compared to the maximum crack width measured using the crack width comparator, as shown in Figure 4.10 and Figure 4.11. For clarity, the shear load versus crack width behavior is only shown for two specimens in each figure; specimens S-120-0.25%-3-R and S-130-0.25%-3-R representing the specimens with the minimum transverse reinforcement ratio for Group 1 and Group 2, respectively, and specimens S-120-0.5%-3-R and S-130-0.5%-3-R, for the specimens with the maximum stirrups.

Currently the ACI 318-08 Building Code limits the width of flexural cracks to an empirical value of 0.016 inches, as described in the Section 10.6 code commentary. This limit has been established in order to control the flexural crack widths and therefore this research will assume 0.016 inches to be the maximum allowable value at the service load level. The code requires that the calculated stresses in the reinforcement at service load be computed based upon the unfactored moment, with the maximum allowable stress in the tension reinforcing bars at service load equal to $2/3$ the yield strength. Since the primary failure mechanism is diagonal tension, then this limit must apply to the stresses in the transverse reinforcement. For the No. 3 stirrups, the measured yield strength is 69 ksi which means that at the service load level, the maximum allowable stress is 46 ksi. Using the allowable steel stress value of 46 ksi in the equation for $V_s$ provided by ACI 318-08 indicates
that the service load level for the specimens with the minimum amount of stirrups is 43.0 and 49.5 kips for Group 1 and Group 2, respectively. For the specimens with the maximum transverse reinforcement ratio, the service load level is 63.8 and 70.3 kips for Group 1 and Group 2, respectively. The service load levels are shown in the figures and it can be seen that the corresponding crack widths for each specimen are still less than 0.016 inches, which is the acceptable limit at service load. The data measured from the Group 3 specimens with an $a/d$ ratio of 1.5 is inconsistent and not representative of the typical cracking behavior due to the dominance of arching action as the primary failure mechanism. The measured crack width for the remaining specimens within Group 1 and Group 2 is presented in Appendix C.
Figure 4.10: Measured crack width for Group 1 specimens
Figure 4.11: Measured crack width for Group 2 specimens

4.2 Load-deflection behavior

The typical curves of the applied shear force versus measured deflections for Group 1 and Group 2 specimens are shown in Figure 4.12 and Figure 4.13, respectively. It can be seen from these curves that the specimens without transverse reinforcement failed shortly after the formation of the first diagonal cracks while specimens with transverse reinforcement
were able to sustain higher load levels. For the beam sets within *Group 1* and *Group 2*, the load-deflection behavior was similar for all specimens.

![Graph showing load-deflection behavior of Group 1 specimens](image)

**Figure 4.12: Load-deflection behavior of Group 1 specimens**
The load at the formation of the first critical diagonal crack was taken as the concrete contribution to the shear resistance, $V_c$, for all specimens within the same group. It can be observed from the figures that by increasing the transverse reinforcement ratio from 0.25% to 0.5%, the shear resistance of the beams was increased. The measured shear at failure for the specimens with shear reinforcement was taken as the nominal shear strength, $V_n$. The steel
contribution to the shear strength, $V_s$, was simply determined as the difference between the two values, $V_n$ and $V_c$.

The load-deflection behavior for specimens within *Group 3* is shown in Figure 4.14. It can be observed from these curves that the load-deflection behavior for this beam set is quite different from that of *Group 1* and *Group 2*, which is to be expected due to the decreased shear span-to-depth ratio. The reduction in $a/d$ from 3.0 to 1.5 caused the formation of a primary diagonal compression strut for all of the test specimens, which caused the “arching action” mechanism to take place within the shorter shear span region. This mechanism added further resistance to the applied load and is believed to be the primary cause of the increased load carrying capacity for the *Group 3* specimens, especially for those without transverse reinforcement, S-130-0%-1.5 (R).
4.3 **Mode of failure**

The failure mode for each group of specimens was highly dependent upon the $a/d$ ratio and the amount of shear reinforcement present. Since *Group 1* and *Group 2* specimens had identical $a/d$ ratios of 3.0, failure for these groups depended primarily on the presence of shear reinforcement. As shown in Figure 4.15, specimens without transverse reinforcement failed shortly after the formation of a single diagonal shear crack that extended from the
support to the location of the applied load. For specimens with shear reinforcement, however, the typical observed failure mechanism consisted of concrete crushing at the nodal zone of the diagonal compression strut located directly below the loading plate under the hydraulic actuator, as shown in Figure 4.16. Failure for these specimens with transverse reinforcement and $a/d$ ratio of 3.0 occurred well after the formation of many diagonal cracks within the shear span.

Figure 4.15: Typical failure mode for specimens without transverse reinforcement ($a/d = 3.0$)
The failure mode of Group 3 specimens was controlled primarily by the spacing of stirrups throughout the entire length of the beam, while arching action within the shorter shear span occurred consistently for all of the specimens. For specimens without stirrups, failure occurred at much higher load levels than were anticipated due to the arching action mechanism within the short shear span. For the two beams with the minimum amount of transverse reinforcement, flexural shear failure was observed in the longer shear span on the
uninstrumented side of the beam when the load carrying capacity of that side was reached as previously shown in Figure 4.5. When the transverse reinforcement ratio was increased to 0.5%, the load carrying capacity of the longer shear span was also increased and failure was governed by arching action within the short shear span of the beam.

![Figure 4.17: Typical failure mode due to arching action](image)
Figure 4.18 shows testing of the first specimen within Group 3 without transverse reinforcement. It is important to note that loading was stopped at high levels in order to prevent yielding of the longitudinal reinforcement prior to testing the other side with the maximum amount of shear reinforcement. The second identical beam was tested with the portion containing the maximum amount of transverse reinforcement first, then the beam was flipped and the portion without stirrups was tested up to failure as shown in Figure 4.19. Failure of this specimen occurred due to crushing of the concrete within the nodal zone of the compression strut, located directly under the applied load. It can be seen in Figure 4.20 that high tensile stresses in the longitudinal reinforcement exerted splitting forces on the end cover of the beam that were high enough to overcome the bond strength, thus leading to splitting of the concrete cover. A comparison of the failure modes for each individual specimen can be found in Appendix A.
Figure 4.18: Testing specimen S-130-0%-1.5
Figure 4.19: Testing of specimen S-130-0%-1.5-R
4.4 Load carrying capacity

Table 4.1 provides the results for all of the tested specimens. The maximum measured loads given in the table are compared to the predicted values according to the ACI 318-08 equations for shear. The table shows the test results compared to the predicted shear capacity based on the measured yield strength of the steel reinforcement and the actual concrete strength on the day of testing. As described in Chapter 3, since the beams were
designed for a shear controlled failure, the longitudinal reinforcement ratio was increased to prevent a premature flexural failure. To account for this increase in flexural reinforcement, the concrete contribution to the shear strength, $V_c$, was predicted using Eq. 2.26 from ACI 318-08, which takes into account the effect of the longitudinal reinforcement ratio, $\rho_w$.

The ratios of the measured to predicted nominal shear capacities for each test specimen are shown in the table. For the beams without transverse reinforcement within Group 1 and Group 2 the mean value for the measured to predicted ratio of the nominal shear strength is 1.01 with a coefficient of variation of 0.10. However, the results for specimens within Group 1 and Group 2 with transverse reinforcement do not match as closely. The mean and standard deviation for the nominal shear strength of these specimens were determined to be 1.30 and 0.05, respectively. The main reason for this contrariety is the inherent conservatism within the ACI code. Since the equation used to determine the shear resistance provided by stirrups is based on a 45 degree truss model, the predicted values of the nominal shear capacity of these members were much higher than those actually measured during testing due to the formation of much flatter crack angles. A similar comparison was made for the Group 3 specimens, but as expected, the measured values for the nominal shear capacity did not correlate as well with the predicted values using the ACI 318-08 shear behavior equations provided in Chapter 11 of the code. Table 4.1 shows the ratios of the measured to predicted values for these specimens based on the ACI 318-08 shear provisions, however the behavior of beams with such a low $a/d$ ratio should be evaluated using the Strut and Tie Method as recommended by ACI and outlined in Appendix A of the code. The
evaluation of the load carrying capacity for the Group 3 specimens using the Strut and Tie model will be presented in Section 4.6.4.
Table 4.1: Evaluation of test results based on ACI 318-08

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Maximum Measured Shear</th>
<th>$f_c$</th>
<th>$V_c$</th>
<th>$V_s$</th>
<th>$V_n$</th>
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<td>Predicted</td>
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<td></td>
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<td>S-120-0%-3</td>
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</tr>
<tr>
<td>S-120-0%-3-R</td>
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</tr>
<tr>
<td>S-120-0.25%-3</td>
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4.5 **Measured strains in the transverse reinforcement**

Stirrup strains were detected using weldable strain gages that were attached to specific stirrups within each beam. The location of the strain gages on the selected stirrups was determined based upon the predicted path that shear cracks would travel within the shorter shear span region during testing. It was anticipated that these cracks, extending from the point of load application to the support, would pass through the stirrups that carried the strain gages. Figure 4.21 and Figure 4.22 show the measured strains obtained from the weldable strain gages for specimens with transverse reinforcement within *Group 1* and *Group 2*, respectively. It can be seen from Figure 4.21 that the initiation of diagonal shear cracks began around a load level of 30 kips for *Group 1*, evident by the measured strains in the stirrups. For *Group 2*, as shown in Figure 4.22, shear cracks formed at a higher load level of around 40 kips when strains were first measured in the stirrups.

Although all of the specimens within each group experienced the onset of shear cracking at almost identical load levels, the cracking behavior of the specimens can be better explained by examining the effect of increasing the transverse reinforcement ratio from 0.25% to 0.5%. Figure 4.21 and Figure 4.22 show that the measured strains in the stirrups at the same load levels were less for the specimens that contained more stirrups within the shear span. When examining the behavior of specimens with the minimum amount of stirrups in Figure 4.21, for example, around the 60 kips applied shear load level the measured strains in
the stirrups were around 0.004 in/in. This indicates that the stirrups had reached their yield plateau, according to the stress-strain relationship shown in Figure 3.11. At the same load level the specimens with the maximum amount of stirrups experienced strains that were around 0.0005 in/in, 87.5% lower, indicating that the stirrups were still within the linear-elastic range. The ability of these specimens to sustain much lower stirrup strains than specimens with the minimum amount of stirrups at the same load level is due to the distribution of the applied load through more transverse steel area within the shear span. The figures also show that although the measured stirrup strains in the beams with the minimum amount of transverse reinforcement were higher than the yield strain, the tested specimens were able to carry increasing loads. This behavior is due to the redistribution of load to different stirrups within the shear span until failure ultimately occurs once all of the stirrups have reached or exceeded their yield stress levels. For the specimens with the maximum amount of stirrups, failure occurred when the strains were at yield, suggesting that redistribution of stresses occurred through stirrups that did not have strain gages attached before those that did have them experienced the full effect of the load. The measured strains obtained from the weldable strain gages within the Group 3 specimens are presented in Appendix D.
Figure 4.21: Stirrup strains for Group 1 specimens
Figure 4.22: Stirrup strains for Group 2 specimens

Specimen S-130-0.5%-R was selected to have multiple weldable strain gages placed within the shear span to obtain a wider range of strain data. This specimen was chosen specifically because it contained the maximum allowable amount of stirrups and had a targeted concrete compressive strength of 4000 psi, which is widely used in most structural applications. Six weldable strain gages were placed along the shear span at varying heights
on the stirrups according to the expected diagonal shear crack as shown in Figure 4.23 and Figure 4.24.

Figure 4.25 shows the measured strains at varying load levels for the stirrups within the shear span of the tested specimen. Strain gages 3 and 5 seem to have similar strain levels at and prior to the ultimate failure load; however strain gage 3 does experience higher strains at the lower load levels due to the close proximity of shear cracks at these times compared to strain gage 5. It can also be observed that strain gages 2 and 6 seem to be increasing at the same rate as the applied load is increased. Failure occurred when all of the strain gages reached their yield strains as shown in the figure, except for strain gage 1 which does not seem to be affected as much by the increase in applied load. This can be attributed to the crack formation away from the strain gage.

![Diagram showing locations of instrumented stirrups](image)

**Figure 4.23: Locations of the instrumented stirrups for specimen S-130-0.5%-3-R**
Figure 4.24: Locations of the strain gages for specimen S-130-0.5%-3-R
4.6 Comparison of LSP concrete shear behavior to various models

Table 4.1 compares the measured load carrying capacities of the tested specimens to those predicted by the ACI 318-08 Building Code provisions for beams subjected to shear. The table shows a good correlation between the tested results and the predicted capacities for Group 1 and Group 2, however Group 3 did not achieve the same results. This is due to the shorter shear span-to-depth ratio, as previously mentioned. These specimens will be
compared to the Strut and Tie method outlined by the ACI 318-08 Building code. To show
the applicability of the most current code provision models to the LSP concrete, the ultimate
capacities of the tested specimens within Group 1 and Group 2 were compared to predictions
using the AASHTO LRFD Bridge Design Specifications, Canadian Standards Association
(CSA) Code, and the Eurocode 2. Equations used to calculate the code predictions can be
found in Section 2.7.

4.6.1 Comparison to AASHTO LRFD Bridge Design Specification (2007)

The ultimate load carrying capacities of the specimens were predicted using the
AASHTO LRFD Bridge Design Specification according to the general procedure outlined in
Article 5.8.4.2 of the code. The test predictions for each group were determined by first
calculating the concrete contribution to the shear resistance $V_c$. For specimens without
transverse reinforcement, this value was determined to be the nominal shear load carrying
capacity $V_n$. The contribution to the shear resistance provided by vertical stirrups spaced at a
distance $s$ within the shear span was determined for specimens with both the minimum and
maximum amounts of transverse reinforcement within each group. The nominal shear load
carrying capacity for these specimens was simply taken as the summation of $V_c$ and $V_s$.

For specimens without transverse reinforcement, the concrete contribution to the
shear resistance $V_c$ was determined using Eq. 2.40 in conjunction with Table 2.3, as specified
in Article 5.8.2.5 of the code using a sectional analysis taken at a distance equal to $d_v$ away
from the support. The crack spacing parameter $s_{xe}$ was used in the table to establish the row where the values of the longitudinal strain at mid-depth, $\varepsilon_x$, could be interpolated. Using Eq. 2.41, the value of $s_{xe}$ was calculated to be 17.8 inches and values for $\theta$ and $\beta$ were interpolated between the rows corresponding to $s_{xe}$ values of 15.0 inches and 20.0 inches. An assumed value for $\varepsilon_x$ was then used to enter the table and interpolate the corresponding crack angle $\theta$ which was used in Eq. 2.40 along with the shear force and moment taken at the section of interest to calculate the actual value for $\varepsilon_x$. This process was repeated until the assumed mid-depth strain matched the calculated strain, and finally the values of $\theta$ and $\beta$ were used to calculate $V_c$ according to Eq. 2.34.

A similar approach was used to determine the shear resistance for specimens with the minimum and maximum amounts of transverse reinforcement using Eq. 2.39 and Table 2.2. Rather than using the crack spacing parameter $s_{xe}$ to interpolate values of $\theta$ and $\beta$, an assumed nominal shear force was first used to calculate the shear stress ratio $\frac{V}{f'c}$. The appropriate row in the table was located where the assumed shear stress ratio was less than or equal to the next largest provided value. Similarly, an initial assumed value of $\varepsilon_x$ was then used to interpolate the value for $\theta$, which was used in Eq. 2.39 along the shear force and moment taken at the section of interest to calculate the actual value for $\varepsilon_x$. Once convergence of the assumed and calculated strains was achieved, the interpolated values for $\theta$ and $\beta$ were used to calculate $V_c$ and $V_s$ using Eq. 2.34 and Eq. 2.38, respectively. The nominal shear resistance $V_n$ was taken as the summation of these two values and then used to calculate the actual shear...
stress ratio. The resulting value then determined a new row for interpolation and the process was repeated until the assumed and calculated values for $\varepsilon_x$ converged once again. Finally, the nominal shear resistance was re-calculated and taken as the predicted value.

The results of the comparison of the measured ultimate loads to those predicted by AASHTO LRFD design equations are presented in Table 4.2 and show a good correlation between the measured and the predicted values. It was noticed that as the transverse reinforcement ratio increases, the value for $\beta$ decreases, thus reducing the concrete contribution to the shear resistance for members with transverse reinforcement. This inherent pattern in the AASHTO LRFD code is representative of the actual shear cracking behavior of reinforced concrete beams with vertical stirrups. At higher load levels, the crack widths will increase resulting in lower interface shear transfer, and consequently, an overall decrease in the concrete contribution to the shear strength. For specimens without transverse reinforcement, the average of the measured to the predicted ratio of the overall shear load carrying capacity is 1.17 with a coefficient of variation of 0.24. For the specimens with transverse reinforcement, the average becomes 1.17 with a coefficient of variation of 0.15. On average for all of the specimens within Group 1 and Group 2, the measured to the predicted ratio of the load carrying capacity is 1.17 with a coefficient of variation of 0.28. The results show that the AASHTO LRFD Bridge Design Specification code can accurately predict the shear capacity of LSP concrete members, both with and without transverse reinforcement.
Table 4.2: Evaluation of test results according to AASHTO LRFD Design Code

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<th>Specimen ID</th>
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<th>$\theta$</th>
<th>$\beta$</th>
<th>$V_c$</th>
<th>$V_s$</th>
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<td></td>
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<td>Measured</td>
<td>Predicted</td>
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<td>38.3°</td>
<td>3.08</td>
<td>31</td>
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<td>-</td>
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<tr>
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<td>29.9°</td>
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<td>1.73</td>
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4.6.2 Canadian Standard Association (CSA)

Much like the AASHTO LRFD code, the shear equations proposed by the Canadian Standard Association (CSA 2004) are based on the Simplified Modified Compression Field Theory, and therefore the results of the two code predictions are quite similar when compared. The general procedure outlined in Section 11.3.6.4 of the CSA code was used to determine the maximum shear load carrying capacity for all of the test specimens and the predictions were compared to the actual measured loads. Just like the procedures of the AASHTO LRFD and ACI 318 codes, the concrete contribution to the shear resistance $V_c$ was first calculated, and for members without transverse reinforcement, was taken as the nominal shear resistance. Next, the steel contribution to the shear resistance provided by vertical stirrups $V_s$ was calculated for members with transverse reinforcement and the nominal shear resistance was taken as the summation of $V_c$ and $V_s$.

Instead of using tables to determine the values for $\theta$ and $\beta$ like the AASHTO LRFD, the CSA code has simplified equations that make it less cumbersome for designers to use. For all of the specimens, an assumed mid-depth strain was first used to calculate the values for $\beta$ and $\theta$ using Eq. 2.47 and Eq. 2.48, respectively. For specimens without transverse reinforcement, the value for $s_{ce}$ was calculated using Eq. 2.49 but was limited to 11.8 inches (300 mm) for specimens with transverse reinforcement. $V_c$ and $V_s$ were then calculated using Eq. 2.45 and Eq. 2.46, respectively and once the nominal shear force was determined, this
value was used in Eq. 2.51 to calculate the actual value for $\varepsilon_x$. This process was continued until convergence of the assumed and actual mid-depth strain had been reached.

Table 4.3 shows the comparison of the measured ultimate loads to those predicted by the CSA design equations. From the table, it can be seen that the CSA code can be used to conservatively predict the shear behavior of beams made with LSP concrete that contain transverse reinforcement, and conversely the results are a bit unconservative for those that do not. Similar to the AASHTO LRFD comparison, it was noticed that as the transverse reinforcement ratio increases, the value for $\beta$ decreases, thus reducing the concrete contribution to the shear resistance for members with transverse reinforcement. For specimens without transverse reinforcement, the average of the measured to the predicted ratio of the overall shear load carrying capacity is 1.08 with a coefficient of variation of 0.25. For the specimens with transverse reinforcement, the average becomes 1.18 with a coefficient of variation of 0.14. On average for all of the specimens within Group 1 and Group 2, the measured to the predicted ratio of the load carrying capacity is 1.14 with a coefficient of variation of 0.31.
Table 4.3: Evaluation of the test results according to CSA

<table>
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<tr>
<th>Specimen ID</th>
<th>Maximum Measured Shear</th>
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<th>$\beta$</th>
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<th>Measured vs Predicted</th>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0%-3</td>
<td>41</td>
<td>6750</td>
<td>30.6°</td>
<td>0.26</td>
<td>40</td>
<td>0.98</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>S-130-0%-3-R</td>
<td>38</td>
<td></td>
<td>32.3°</td>
<td>0.24</td>
<td>36</td>
<td>1.09</td>
<td>57</td>
<td>45</td>
<td>1.27</td>
<td>81</td>
<td>1.20</td>
<td>1.02</td>
<td>0.94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0.25%-3</td>
<td>97</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0.25%-3-R</td>
<td>104</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0.5%-3</td>
<td>130</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0.5%-3-R</td>
<td>132</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>
4.6.3 Eurocode 2

Results for all of the test specimens were predicted using the Eurocode 2 which is based on a variable angle strut model where the angle of strut inclination for members with transverse reinforcement may be varied within certain limitations to provide the required capacity of the beam. Unlike the previously compared code predictions, the Eurocode does not account for $V_c$ when determining the nominal shear capacity of beams that contain stirrups, but instead considers only the minimum value of $V_s$. The angle of inclination of the compression strut is calculated for these members, however it is only used in determining $V_s$, and therefore the variable $\theta$ is not determined for members without shear reinforcement.

A comparison of the measured ultimate loads to those predicted by the Eurocode 2 is shown in Table 4.4. The nominal shear capacities for specimens without transverse reinforcement were determined using Eq. 2.52, where $V_c$ is simply a function of the concrete strength, longitudinal reinforcement ratio and the physical dimensions of the cross sectional area. For members with transverse reinforcement, an assumed value for $\theta$ was used to calculate $V_s$, taken as the minimum of Eq. 2.54 and Eq. 2.55, which was substituted into the following equation to obtain the actual angle of the concrete compressive strut:
\[ \theta = 0.5 \sin^{-1} \left( \frac{v}{0.20 f'_c (1 - f'_c/250)} \right) \]  

Eq. 4.2

where:

\( v \) is the shear stress ratio, taken as \( \frac{V_s}{b_w d} \), MPa;

This process was repeated until the assumed value for \( \theta \) converged with the value calculated from Eq. 4.2.

Since the Eurocode does not account for the concrete contribution to the shear capacity for members that contain transverse reinforcement, the level of conservatism of the code predictions is increased which can be seen in the results shown in Table 4.4. The table shows a fair amount of conservatism for the members without transverse reinforcement, with an average of 1.2 and a coefficient of variation of 0.22. For the specimens with transverse reinforcement, the average becomes 1.25 with a coefficient of variation of 0.34. The overall average of the measured to the predicted ratio of the load carrying capacity for all of the specimens within Group 1 and Group 2 is 1.23 with a coefficient of variation of 0.41.
Table 4.4: Evaluation of the test results according to Eurocode 2

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Maximum Measured Shear</th>
<th>( f_c )</th>
<th>( \theta )</th>
<th>( V_c )</th>
<th>( V_s )</th>
<th>( V_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kips</td>
<td>psi</td>
<td></td>
<td>Measured</td>
<td>Predicted</td>
<td>Measured vs Predicted</td>
</tr>
<tr>
<td><strong>Group 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-120-0%-3</td>
<td>35</td>
<td>-</td>
<td>21.8(^\circ)</td>
<td>39</td>
<td>30</td>
<td>1.29</td>
</tr>
<tr>
<td>S-120-0%-3-R</td>
<td>43</td>
<td>-</td>
<td>29.7(^\circ)</td>
<td>4150</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-120-0.25%-3</td>
<td>93</td>
<td>-</td>
<td>21.8(^\circ)</td>
<td>44</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-120-0.25%-3-R</td>
<td>83</td>
<td>-</td>
<td>29.7(^\circ)</td>
<td>87</td>
<td>98</td>
<td>0.88</td>
</tr>
<tr>
<td>S-120-0.5%-3</td>
<td>126</td>
<td>-</td>
<td>29.7(^\circ)</td>
<td>85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-120-0.5%-3-R</td>
<td>124</td>
<td>-</td>
<td>29.7(^\circ)</td>
<td>6750</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Group 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0%-3</td>
<td>41</td>
<td>-</td>
<td>21.8(^\circ)</td>
<td>57</td>
<td>70</td>
<td>0.81</td>
</tr>
<tr>
<td>S-130-0%-3-R</td>
<td>38</td>
<td>-</td>
<td>21.8(^\circ)</td>
<td>64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0.25%-3</td>
<td>97</td>
<td>-</td>
<td>21.8(^\circ)</td>
<td>90</td>
<td>127</td>
<td>0.71</td>
</tr>
<tr>
<td>S-130-0.25%-3-R</td>
<td>104</td>
<td>-</td>
<td>21.8(^\circ)</td>
<td>92</td>
<td>127</td>
<td>0.72</td>
</tr>
<tr>
<td>S-130-0.5%-3</td>
<td>130</td>
<td>-</td>
<td>23.9(^\circ)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0.5%-3-R</td>
<td>132</td>
<td>-</td>
<td>23.9(^\circ)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.6.4  The Strut and Tie Method

The strut and tie method was used to model the beam specimens in Group 3 with a shear span-to-depth ratio of 1.5. The mechanics of this method, described in Section 2.6.4, allowed the flow of compressive stresses throughout the beams to be carried by diagonal concrete struts, while the tensile stresses were carried exclusively by the tension reinforcement. The strut and tie model created for the analysis of the Group 3 specimens is shown in Figure 4.26, and this model was used to predict the behavior for each of the specimens within the group.

Figure 4.26: Strut and Tie model for Group 3 specimens
The location of the horizontal tension tie was chosen based upon the location of the tension reinforcement at a distance from the extreme compression fiber, symbolized as the variable $d$. Since $d$ is located at 14.4 inches from this location, the centroid of the tension tie was determined to be 3.6 inches from the extreme tension fiber. The 8 inch wide bearing plates provided the base width for each of the three nodal zones shown in the figure and the depth of the two nodal zones located on the bottom was equal to twice the vertical distance to the tension tie. This distance was chosen because the tensile force created by sufficient bond development of the tension reinforcement through hooked bars must act at the centroid of the nodal area. The vertical reactions located at the center of the bearing plates provided the horizontal location of each bottom node, determined to be a C-C-T classification through static equilibrium. The two compression struts were designed to be tapered, with larger areas at the bottom nodes than at the top.

The capacity of the model was calculated as specified in the ACI 318-08 Building Code, Appendix A without the use of reduction factors. The calculations showed that the controlling failure mechanism would be crushing of the top node depending on the total depth of the compression zone. The depth of the top node, and consequently the compression zone, was adjusted until both of the top strut-node interface stresses reached the limit of $0.85f'_c$ at exactly the same time. The vertical components of each strut were combined to form the maximum applied load of 284 kips, resulting in a maximum force of 190 kips throughout the shear span according to the model. The tension tie force was then determined through static equilibrium to be 336 kips, resulting in a tensile stress of 56 ksi.
In order for the yield stress to be developed in the tension tie, sufficient development length must be provided through mechanical anchorage devices, standard hooks, or straight bar development as per section 12.5.1 of the ACI 318-08 Building Code. The development length, \( \ell_{dh} \), for the six deformed bars terminating in standard 90 degree hooks was determined to be 20 in. according to the following equation provided by section 12.5.2 of the code:

\[
\ell_{dh} = \left( 0.02\psi_e \frac{f_y}{\lambda \sqrt{f'_{c}}} \right) d_b
\]

\textbf{Eq. 4.3}

where:

- \( \psi_e \) is a factor taken as 1.2 for epoxy-coated reinforcement, otherwise taken as 1.0
- \( f_y \) is the specified yield strength of reinforcement, psi
- \( \lambda \) is the lightweight concrete modification factor taken as 1.0 for normal-weight concrete as provided in section 8.6.1 of ACI 318-08.
- \( f'_{c} \) is the specified concrete compressive strength, psi

The distance from the end of the 90 degree hooks to the point where the tension tie leaves the bottom left extended nodal zone, denoted as \( \ell_{anc} \), is equal to 16.62 inches. Section A.4.3.1 of the code states that “Nodal zones shall develop the difference between the tie force on one side of the node and the tie force on the other side.” This indicates that the available tension tie stress able to be developed at the distance \( \ell_{anc} \) is equal to 61 ksi, as shown in Figure 6.8 of Appendix E. Since the tension tie stress determined through equilibrium is less than this value, then it is appropriate to conclude that the tension steel is not yielding and that
crushing of the top node is the controlling limit state for the strut-and-tie model. This conclusion is also upheld by examining that the bearing stress at each support is below the crushing stress limit of $0.85f'_c$. Detailed results of the strut-and-tie analysis are shown in Appendix E.

During the testing of specimen S-130-0%-1.5 without transverse reinforcement, the loading was stopped at a high level to prevent yielding of the longitudinal reinforcement prior to testing the other side of the beam with the maximum amount of transverse reinforcement. For the second specimen without transverse reinforcement, S-130-0%-1.5-R, the side with the stirrups was tested first and then the beam was rotated to test the side without stirrups. The maximum measured shear load was 206 kips which gives a measured to predicted ratio of 1.08.

For the specimens with the minimum amount of stirrups, failure occurred on the side of the beam with a shear span-to-depth ratio of 3.0. The measured shear load on this respective side was 97 kips, similar to specimen S-130-0.25%-3 from Group 2. The beam was severely damaged from the first test and when it was rotated to test the second specimen the failure of the first test affected the behavior of the second test, thus the results were discarded.

For the specimens with the maximum transverse reinforcement ratio, the maximum measured loads were 214 kips for specimen S-130-0.5%-1.5 and 215 kips for specimen S-130-0.50%-1.5-R. These results show the highest measured-to-predicted ratios from the
entire group equal to 1.13 since the STM method provides upper-bound values for predicting the shear capacity.

Table 4.5: Evaluation of the test results for Group 3 according to the Strut-and-Tie Model

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Maximum Measured Shear</th>
<th>$f_c$</th>
<th>Maximum Predicted Shear</th>
<th>Measured vs Predicted Shear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kips</td>
<td>psi</td>
<td>kips</td>
<td></td>
</tr>
<tr>
<td>S-130-0%-1.5</td>
<td>153</td>
<td>6970</td>
<td>190</td>
<td>0.81</td>
</tr>
<tr>
<td>S-130-0%-1.5-R</td>
<td>206</td>
<td></td>
<td></td>
<td>1.08</td>
</tr>
<tr>
<td>S-130-0.25%-1.5</td>
<td>194</td>
<td></td>
<td></td>
<td>1.02</td>
</tr>
<tr>
<td>S-130-0.25%-1.5-R</td>
<td>159**</td>
<td></td>
<td></td>
<td>0.84</td>
</tr>
<tr>
<td>S-130-0.5%-1.5</td>
<td>214</td>
<td></td>
<td></td>
<td>1.13</td>
</tr>
<tr>
<td>S-130-0.5%-1.5-R</td>
<td>215</td>
<td></td>
<td></td>
<td>1.13</td>
</tr>
</tbody>
</table>

* Testing was stopped early before failure occurred.
** Specimen was unable to achieve high loads due to the previous failure in the longer shear span
5.0 CONCLUSIONS AND RECOMMENDATIONS

5.1 Overview

This research investigated the shear behavior of reduced unit weight reinforced concrete beams with lightweight synthetic particles (LSP). LSP is an admixture consisting of smooth-skinned, closed-cell particles that reduce the unit weight of reinforced concrete members when incorporated into the concrete mix design process. This characteristic of LSP concrete could reduce the overall cost of construction for future structures. Other advantages of using the LSP include enhanced flowability of the fresh concrete for pumping purposes, increased durability for freeze/thaw and deicing exposed conditions, and a reduction in thermal conductivity of concrete, thus reducing the energy required for heating and cooling of buildings. Complementary research at North Carolina State University has also investigated the bond behavior of reinforced concrete beams using LSP (Hosny, 2010), however this research focused entirely on the shear behavior.

The research included an experimental program conducted at the Constructed Facilities Laboratory, North Carolina State University, consisting of nine reinforced concrete beams which were all fabricated on-site with LSP concrete. Testing was performed using a single point load located closer to one end of each beam, allowing for a total of eighteen tests to be completed. The main parameters considered in the study included the concrete unit weight, concrete compressive strength, the amount of transverse reinforcement, and the shear span-to-depth ratio \(a/d\). The measured data collected from the tested specimens, concrete
cylinders and steel reinforcement were used to analyze and evaluate the overall shear behavior. Test results were compared to predicted values using four design codes including the strut-and-tie analysis.

5.2 Conclusions

The research findings of the experimental program conducted specifically to study the shear behavior of reduced unit weight concrete using lightweight synthetic particles resulted in the following conclusions:

1. The concrete containing the LSP additive used in this investigation, with unit weights ranging between 120 and 130 lb/ft$^3$, achieved comparable compressive strength to what is commonly used for structural applications.

2. Concrete containing LSP additive was flowable and had good workability.

3. The load-deflection characteristics and the crack pattern of the beams tested in this experimental program were similar to the expected behavior of beams using normalweight concrete without LSP.

4. Cracked section analysis used to predict the flexural and shear behavior of regular concrete could be used to predict the behavior of concrete members produced with LSP additive.

5. The beams with the LSP concrete additive satisfy the shear design requirements of the ACI 318-08 building code.
6. The ACI, CSA, AASHTO LRFD and Eurocode 2 can conservatively predict the shear behavior of concrete members produced with LSP additive.

7. Test results also confirmed that, for structural design of LSP concrete containing normal weight aggregates, the reduction factor $\lambda$ normally used for the lightweight concrete, need not be applied for LSP concrete with unit weight equal to or higher than 120 lb/ft$^3$.

5.3 **Recommendations for Future Research**

Based on the results and conclusions of this research program, several recommendations are provided for future research on the shear behavior of LSP concrete structural members.

- The comparison made between the results from the experimental program and the predictions of the ACI 318-08 Building code indicates that shear design of structural members can be accomplished with a certain amount of conservatism. It is recommended that future research be conducted with modified testing parameters to add more test specimens and justify confidence of this conclusion.

- A research area that could greatly benefit from reduced unit weight concrete using the LSP additive is that of precast, prestressed concrete construction. It is recommended that a future research program test the shear behavior of prestressed concrete using lightweight synthetic particles. Results from such a program could permit the use of
LSP concrete in prestressing applications and could potentially reduce the cost of shipping precast concrete members to the construction site.

- Paired with a prestressed LSP concrete experimental program, an investigation on the bond mechanism of LSP concrete to prestressing strands along with the effect of transfer and development length would be beneficial for future use in this area, as these mechanisms have a significant impact on prestress losses.

- It is recommended that a creep and shrinkage study be conducted to determine the long-term behavior of LSP concrete under sustained load and the effect of different LSP percentages in the mix design. This information would be very beneficial for prestressed concrete applications.

- It is recommended that future research on the topic of shear using LSP concrete be expanded into the punching shear behavior of slabs. An in-depth study into the punching shear failure of LSP concrete slabs could be very useful and ultimately reduce the weight and cost of structural slab-type members.
REFERENCES


APPENDICES
APPENDIX A

Failure Comparisons

(a) S-120-0.25%-3
(b) S-120-0.25%-3-R

(c) S-120-0.5%-3
(d) S-120-0.5%-3-R

(e) S-120-0%-3
(f) S-120-0%-3-R

Figure 6.1: Failure comparison of Group 1 specimens
Figure 6.2: Failure comparison of Group 2 specimens
Figure 6.3: Failure Comparison of Group 3 specimens
## APPENDIX B

### Average Compressive Strength of Concrete

**Table 6.1: Average compressive strength of test specimens**

<table>
<thead>
<tr>
<th>Specimens</th>
<th>7 Days</th>
<th>14 Days</th>
<th>28 Days</th>
<th>Day of Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-120-0%-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-120-0.5%-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-120-0%-3-R</td>
<td>2640</td>
<td>3140</td>
<td>2760</td>
<td>4270</td>
</tr>
<tr>
<td>S-120-0.5%-3-R</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-120-0.25%-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-120-0.25%-3-R</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0%-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0.5%-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0%-3-R</td>
<td>4760</td>
<td>5590</td>
<td>5930</td>
<td>6860</td>
</tr>
<tr>
<td>S-130-0.5%-3-R</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0.25%-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0.25%-3-R</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0%-1.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0.5%-1.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0%-1.5-R</td>
<td>4760</td>
<td>5590</td>
<td>5930</td>
<td>6953</td>
</tr>
<tr>
<td>S-130-0.5%-1.5-R</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0.25%-1.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-130-0.25%-1.5-R</td>
<td></td>
<td></td>
<td></td>
<td>7068</td>
</tr>
</tbody>
</table>
APPENDIX C

Measured Crack Width

Figure 6.4: Measured crack width for remaining Group 1 specimens ($\rho_i > 0\%$)
Figure 6.5: Measured crack width for remaining Group 2 specimens ($\rho_t > 0\%$)
Figure 6.6: Stirrup strains for Group 3 specimens
APPENDIX E

Strut and Tie Model

Figure 6.7: Strut and tie model for Group 3 specimens
Figure 6.8: Available tension tie stress development at bottom left node

6.3.1 Model Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>3.97 in.</td>
</tr>
<tr>
<td>( f'_c )</td>
<td>6,970 psi.</td>
</tr>
<tr>
<td>0.85( f'_c )</td>
<td>5,925 psi.</td>
</tr>
<tr>
<td>( A_s )</td>
<td>6.00 in(^2)</td>
</tr>
<tr>
<td>( f_y )</td>
<td>74,000 psi.</td>
</tr>
<tr>
<td>( d )</td>
<td>14.40 in.</td>
</tr>
<tr>
<td>( \ell_{dh} )</td>
<td>20.00 in.</td>
</tr>
<tr>
<td>( \ell_{anc} )</td>
<td>16.63 in.</td>
</tr>
</tbody>
</table>
6.3.2 Model Calculations

### Left Strut

<table>
<thead>
<tr>
<th>( A_{cs,\text{top}} = \left[ \left( 4'' + \frac{c}{2} \tan 61^\circ \right) \sin 29^\circ + \frac{c}{2} \cos 29^\circ \right] 12'' )</th>
<th>( = 65.12 \text{ in}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{s,\text{max}} = 0.85f_c A_{cs,\text{top}} )</td>
<td>( = 385.81 \text{ kips} )</td>
</tr>
</tbody>
</table>

### Right Strut

<table>
<thead>
<tr>
<th>( A_{cs,\text{top}} = \left[ \left( 4'' + \frac{c}{2} \cot 16^\circ \right) \sin 16^\circ + \frac{c}{2} \cos 16^\circ \right] 12'' )</th>
<th>( = 58.92 \text{ in}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{s,\text{max}} = 0.85f_c A_{cs,\text{top}} )</td>
<td>( = 349.13 \text{ kips} )</td>
</tr>
</tbody>
</table>

### Loads

<table>
<thead>
<tr>
<th>( P = F_{s,\text{max Left}} \sin 29^\circ + F_{s,\text{max Right}} \sin 16^\circ )</th>
<th>( = 284.38 \text{ kips} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 = \frac{2P}{3} )</td>
<td>( = 189.59 \text{ kips} )</td>
</tr>
<tr>
<td>( R_2 = \frac{P}{3} )</td>
<td>( = 94.79 \text{ kips} )</td>
</tr>
<tr>
<td>( T = F_{s,\text{max Left}} \cos 29^\circ )</td>
<td>( = 336.01 \text{ kips} )</td>
</tr>
</tbody>
</table>
## Stresses

<table>
<thead>
<tr>
<th>Type</th>
<th>Formula</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{b,\text{Top}} )</td>
<td>( \frac{P}{12 \times 8} )</td>
<td>2.96 ksi</td>
</tr>
<tr>
<td>( \sigma_{b,\text{Left}} )</td>
<td>( \frac{R_1}{12 \times 8} )</td>
<td>1.97 ksi</td>
</tr>
<tr>
<td>( \sigma_{b,\text{Right}} )</td>
<td>( \frac{R_2}{12 \times 8} )</td>
<td>0.99 ksi</td>
</tr>
<tr>
<td>( f_s )</td>
<td>( \frac{T}{A_s} )</td>
<td>56.00 ksi</td>
</tr>
</tbody>
</table>