This research presents the behavior of proposed 3-D glass fiber reinforced polymer (GFRP) pultruded sandwich panels designed to enhance the structural efficiency of and to overcome delamination problems typically exhibited by traditional FRP panels. The sandwich panels consist of GFRP laminate plates at the top and bottom, separated by a polyurethane foam core, and connected by through-thickness fibers to achieve composite action. The use of the through-thickness fibers prevents delamination-type failures, increases the out-of-plane properties of the panels, allows low cost manufacturing, and ensures full utilization of the individual material strengths.

The fundamental material characteristics of the sandwich panels are evaluated in three phases. The first phase evaluates the in-plane tensile properties of the GFRP laminate face sheets to determine the effects, if any, of the through-thickness fiber insertion pattern and test direction. The second phase investigates the shear behavior of the tested panels in order to evaluate the effects of various parameters, including through-thickness fiber insertion pattern, corresponding fiber insertions per square inch (fipsi), and testing direction (parallel or perpendicular to pultrusion direction). The third phase focuses on the flexural behavior of the sandwich panels in order to evaluate the effects of the same parameters in addition to the effects of varying panel widths and span lengths. The analytical phase of the research investigation incorporates the measured tensile and shear material characteristics in conjunction with Elementary and Advanced Sandwich Theories to predict the flexural behavior of the 3-D GFRP panels. Based on these
research findings, recommendations are proposed to the manufacturer and design engineers planning to use these sandwich panels in structural applications. The current research has shown that these panels are a viable, cost-effective alternative to conventional construction materials which can be customized for various applications, such as pedestrian bridge decks, construction and industrial mats, truck trailer or rail car components, and marine environment applications.
FUNDAMENTAL CHARACTERISTICS OF 3-D GFRP PULTRUDED SANDWICH PANELS

by

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DEDICATION

to The Patricks....
BIOGRAPHY

Jason Fredrick Patrick was born to Fredrick Allen Patrick, Civil Engineer and Architect, and Jan Elizabeth Kerchenfaut, Registered Nurse, on April 7, 1982 in Mammoth Lakes, California. He is three years older than his brother, Ian William Patrick, and five years older than his sister, Nicole Jean Patrick, both of whom were also born in California. In 1989, his family moved to Greensboro, North Carolina in search of a new life on the east coast. As a child, Jason always spoke of how he would one day attend North Carolina State University. It was of no surprise when in August 2000, he began his first semester at N.C. State in Civil Engineering. He worked extremely hard throughout the next four years to maintain academic perfection. In addition, he undertook several extracurricular activities in Civil Engineering. In 2003, he led the NCSU Concrete Beam Team to a 2nd Place regional finish in the ASCE Carolina’s Conference. In 2004, Jason captained the NCSU Steel Bridge Team and accomplished yet another 2nd Place Regional finish in the ASCE competition. While early career goals were oriented towards being a structural engineer at a design firm, research experience under the supervision of Dr. Amir Mirmiran led Jason to pursue a new path. His involvement with various research projects at the Constructed Facilities Laboratory on NCSU’s Centennial Campus fueled his desire to become an esteemed member of the academic community. On December 15, 2004, Jason graduated Valedictorian of Civil Engineering and delivered a humble yet inspiring speech to his fellow classmates. In the spring of 2005, he began his graduate studies in pursuit of a Master of Science degree under the direction of Dr. Sami Rizkalla. Upon completion of this thesis, Jason desired to pursue knowledge in an even more challenging quest. In the fall of 2007, he will begin his PhD studies in the realm of Computational Mechanics under the direction of Dr. Arif Masud at the University of Illinois at Urbana-Champaign, also his father’s alma mater. Jason’s pursuit for and the advancement of scientific knowledge is his reason for existence. There are very few things which can compare to the pleasure that engineering and mathematics brings him. In closing, here is a quote from Jason’s Valedictory address: “There truly is a difference in knowing the path and walking the path.”
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CHAPTER 1

1 INTRODUCTION

1.1 INTRODUCTION

As architects continue to expand the envelope of conventional design, engineers must meet stricter demands for structural efficiency in their practice. The research and development of new, innovative materials allows for these realizations to not only be met but also surpassed. However, the development of pioneering materials also brings new challenges for engineers desiring to implement them in structural practice. In the past century, sandwich panels have made their way into various aerospace, construction, marine, and transportation industries. The classical sandwich panel consists of a low density core material “sandwiched” and adhesively bonded between two thin, stiff face sheets. Sandwich panels are analogous to steel I-sections where the outer face sheets act like the flanges to carry tensile and compressive loads whereas the core mimics the web in providing shear resistance. Because the outer face sheets or skins typically have a higher flexural stiffness than the core, placing them farther apart results in enhanced utilization of the individual material components. In general, the faces and adhesive joints can be comprised of various materials, all depending on the requirements of the structure and capabilities of the manufacturing process. The availability of materials is vast and since the introduction of Fiber Reinforced Polymers, the number of choices continues to grow. This unbounded growth may appear as an additional complexity but, in reality, it is one of the main advantages of sandwich construction. The varying
geometry and immense number of material choices for sandwich panels results in designs that best utilize the various properties for an intended purpose.

1.2 RESEARCH OBJECTIVE

Previous research conducted at North Carolina State University’s Constructed Facilities Laboratory showed that various parameters affect the material behavior of the proposed 3-D GFRP sandwich panels. These include fiber insertion density, fiber insertion pattern and test direction, panel thickness, and GFRP laminate skin thickness (Reis, 2005). The focus of this research is to investigate the fundamental structural characteristics of the proposed 3-D GFRP sandwich panels based on an extensive experimental program for 2 in. thick panels consisting of 4 ply laminate skins. The experimental research program is divided into three phases. The first phase is to determine in-plane tensile material characteristics of the GFRP laminate skins with varying fiber insertion patterns and test direction in an effort to obtain useful data to assist in the analysis and prediction of flexural behavior. The focus in phase two is to determine the shear material characteristics of the core with varying fiber insertion patterns and test direction. Since sandwich panels are highly susceptible to shear deformation, the core shear characteristics will also be necessary to analyze to predict the flexural behavior. Phase three focuses on the flexural response of simply supported members subjected to 3-point bending with varying span lengths, fiber insertion patterns and direction, and panel widths. These tests will be used to isolate flexural parameters and verify analytical models.
The analytical phase of this research program involves the application of Elementary Sandwich Theory (EST) and Advanced Sandwich Theory (AST) to predict the flexural behavior of the 3-D GFRP innovative panels for varying span lengths. Elementary Sandwich Theory is similar to simple beam theory with the addition of a shear deformation term and is often referred as “Timoshenko Beam Theory”. Advanced Sandwich Theory also considers both bending and shear deformation but, in addition, realizes the face sheets must bend locally in order to follow the shear deformation of the core. Thus additional shear deflections are reduced by the local bending stiffness of the faces. The analytical phase is also used to provide designers with engineering guidelines and limitations.

1.3 SCOPE AND CONTENT

This research investigation in this thesis is subdivided into two phases: experimental and analytical studies. The experimental phase consists of face sheet tensile behavior, core shear behavior, and flexural behavior of the sandwich panels. The analytical phase is based on Elementary and Advanced Sandwich Theories to predict flexural response through a piecewise linear implementation in MATLAB.

Chapter 2 is a review of the history of sandwich panels, various applications, and analytical models developed. In addition, the results of previous research conducted on similar 3-D GFRP sandwich panels are discussed.

Chapter 3 provides a detailed description of the experimental methodology used to determine the various tensile, shear, and flexural characteristics of the proposed 3-D GFRP sandwich panels.
Chapter 4 presents the results obtained from the various experimental procedures carried out on the sandwich panels. A discussion of the research findings is presented. In addition, a section concerned with manufacturing related issues observed throughout the experimental program is included.

Chapter 5 describes the analytical models and procedure used in the prediction of the flexural behavior. In addition, conclusions reached and recommended engineering design limitations are presented.

1.4 ACKNOWLEDGEMENTS
I would first like to thank my graduate advisor, Dr. Sami Rizkalla, for his patience, direction, and enduring support throughout my graduate studies. In addition, I want to thank Dr. Vernon Matzen and Dr. Paul Zia, my thesis committee members, for their guidance and friendship over the course of my college career in Civil Engineering. I am also ever grateful for the prior knowledge, experience, and assistance provided by Dr. Engin Reis in the early phases of the research project. I am fortunate to have been able to perform experimental research under at the ICBO accredited Constructed Facilities Laboratory (CFL) located on North Carolina State University’s Centennial Campus. This laboratory is a result of the joint venture between the National Science Foundation (NSF) and the University Cooperative Research Center on Repair of Buildings and Bridges with Composites (RB²C). I would like to thank Martin Marietta Composites, a member of RB²C, for providing the research funding and abundance of test specimens. It was a pleasure working with Wes Ballew, the design engineer and representative for Martin Marietta Composites. I appreciate not only his patience but
willingness to provide me with insight into the manufacturing process of the panels. In addition I would like to acknowledge the following people for their contributions to this project’s success:

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Terry Briggs – Machining and welding of experimental test setup components
Rick Lamy – Machining and welding of experimental test setup components
CHAPTER 2

2 LITERATURE REVIEW

2.1 HISTORY & APPLICATIONS OF SANDWICH PANELS

The concept of sandwich construction has been considered by many inventors and engineers at various times in history. Some of the earliest sandwich panel drawings can be found in the works of the ubiquitous Leonardo da Vinci. However, the first person to formally describe and record the concept is accredited to a Frenchman, Duleau, in 1820, and later by Sir William Fairbairn in 1849 (Zenkert, 1997). In 1919, the first sandwich panel was fabricated using thin walled mahogany skins bonded to an end-grain balsa wood core. An end-grain shape means the balsa wood is cut up into cubic pieces and bonded together so that a block is produced where the fiber direction is perpendicular to the plane of the block. With this procedure several advantages were gained; the principal direction of stiffness is perpendicular to the faces and humidity, which is primarily spread along the fibers, would merely cause localized damage. The balsa core panel was used as the primary structural component of the pontoons of seaplanes. However, it was the invention and widespread acceptance of structural adhesives in England and the United States in the 1930’s that allowed the mass production of bonded sandwich composites. By World War II, the military began to use plywood veneer facings bonded to balsa wood cores in various structural components of aircraft. The famous “Mosquito” bomber, produced by the de Havilland Airplane Company, used these sandwich panels in parts of the airframe. The excellent performance of this airplane led to the acceptance by many designers of the superiority of the sandwich panels as an efficient structural
member (Allen, 1969). In 1945, the first all aluminum sandwich panel was manufactured. The panel was composed of thin aluminum facings bonded to an aluminum honeycomb core. Honeycomb consists of an array of open cells, formed from very thin sheets of material attached to each other. Usually the cells form hexagons, but other cell configurations are also used, as shown in Figure 2.1. Paper versions of the honeycomb core had been developed earlier for use in furniture built by Lincoln Industries in the late 1930s. The new aluminum honeycomb system was made possible with the development of better structural grade adhesives which were higher in viscosity and would not simply drip down the cell walls (Bitzer, 1997).

The late 1950’s and early 1960’s brought about the advent of poly-vinyl chloride (PVC) and polyurethane (PUR) core materials. Although PVC foams were developed in Germany in the early 1940’s they were not commercially utilized until later decades due to their softness. Cellular foams did not offer the same high stiffness and strength to weight ratios as honeycombs but had other very important advantages. Cellular foams in general are less expensive than honeycombs, surface preparation and shaping of the core is simple, and the foam surface is easier to adhesively bond to. In addition, cellular foams offer high thermal insulation, acoustical damping, and the closed cell structure ensures buoyancy and prevents water penetration. Polyurethane foams can also be made fire resistant by using additives containing phosphorus (Zenkert, 1997). The next generation of core materials under development is cellular thermoplastic cores where properties can be tailored by varying the orientation of the cell structure.

Sandwich panels designed for aircraft structures almost always employ metal faces with metal honeycomb or corrugated cores. Smooth metallic skins are desirable for
low wind resistance while the honeycomb core offers excellent fatigue characteristics. Panels used in construction related industries have evolved to consist of various materials for one main reason: cheaper production costs. Non-metallic face sheet materials include asbestos cement, plywood, plasterboard, resin-impregnated paper, and fiber reinforced polymer composites (FRP). Non-metallic honeycomb and corrugated cores are typically made of resin impregnated papers and fiber reinforced plastics. Commonly used solid core materials are balsa wood, chipboard, expanded plastics, lightweight concrete, and polyurethane foams. In addition to having a high strength to weight ratio, sandwich panels offer exceptional insulation and acoustical characteristics. A further breakdown of the applications of various sandwich material combinations and designs are listed below:

**Aerospace Industry:** Sandwich composites are increasingly being used in the aerospace industry because of their high strength to weight ratio and enhanced fatigue resistance. Floorboards, composite wings, horizontal stabilizers, composite rudders, landing gear doors, speed brakes, flap segments, aircraft interiors and wingspans are typically made of sandwich composites.

**Marine Industry:** Sandwich composites are well suited for the marine industry’s most demanding designs, including stealth naval vessels. Foam cores meet the critical requirements of strength, buoyancy and low water absorption while non-metallic skins offer high corrosion resistance. Applications include the construction of bulkheads, hulls, decks, and furniture (Allen, 1998).

**Transportation Industry:** High strength-to-weight ratios of sandwich composites offer great advantages to the transportation industry. The insulating, sound damping, and
low cost characteristics make them an appropriate choice for walls, floors, doors, panels and roofs for vans, trucks, trailers and trains.

**Architectural Industry:** Foam cores provide excellent thermal and acoustical insulation which makes them an attractive option for the architectural industry. Typical applications include portable buildings, office partitions, roof panels, building facades, and freezer liners (Koschade, 2002).

Research into the theoretical analysis of sandwich constructions began following World War II. Several papers were published between 1945 and 1955 on the strength and stability of sandwich beams, columns, and plates. A considerable portion of the early research into sandwich panels was undertaken by the Forest Product laboratories of the United States Forest Service. The published papers were both experimental and analytical in nature. Also during this period, Reissner published his well known theory on sandwich plates in which he derives the differential equation for the deflection of a sandwich panel. Libove and Batdorf derived differential equations for the shear forces and deflections of orthotropic panels with thin faces. Hoff, considering the strain energy of a sandwich panel, derived the governing differential equation for an isotropic panel with respect to thick faces. Mindlin derived the governing equation of motion for an isotropic plate accounting for both transverse shear deflections and rotational inertia. These theories provided the basis of the two important books on sandwich constructions published in the 1960’s by Plantema and Allen (Zenkert, 1997).

In the last thirty years the emphasis on theoretical research has shifted more to the optimization of sandwich structures. Finite element techniques, incorporating specially designed sandwich elements, allow for the accurate analysis of sandwich design
problems. The FEA methods are generally more accurate than many of the existing analytical solutions which require several approximations and the use of the finite difference methods to solve the differential equations. Research over the last two decades has revolved primarily around the areas of impact resistance, fatigue, and fracture analysis.

2.2 FUNDAMENTALS OF A SANDWICH ELEMENT

As illustrated in Figure 2.2, a typical sandwich element consists of three main components: two thin, strong face sheets adhesively bonded to a thick, weaker core. The adhesive layer between the core and face sheet interface serves as a load transfer mechanism through the individual components. Placing the stiffer face sheets further apart by use of the greater core thickness increases the overall sandwich element’s moment of inertia, thus increasing the panel’s flexural rigidity.

The face sheets’ main purpose is to carry normal tensile and compressive stresses induced in the sandwich element. These “skins” are typically constructed of high modulus materials such as steel, aluminum, titanium, glass, aramid, or carbon Fiber Reinforced Polymer (FRP) composites. The deformation associated with these members is primarily due to bending with negligible amounts of contributing shear deformation.

The core’s main function is to separate the face sheets at a constant distance from each other in an effort to maximize the moment of inertia for the section. In addition, the core should be able to provide enough resistance to the induced shear stresses. Furthermore, the core should be stiff enough to keep the face sheets flat so as to prevent them from buckling out of plane under induced compressive forces. In the flexural
behavior of the steel I-beam, in the preceding analogy, the web provides relatively high resistance to shear forces and thus experiences low and often negligible amounts of shear deformation. In the case of sandwich panels, where the core is often weak relative to the face sheets, shear deformation of is often substantial and should be accounted for. The qualitative effect of shear rigidity is illustrated in Figure 2.3. Figure 2.3a illustrates a core which is relatively rigid in shear, forcing the face sheets to deform in unison. Figure 2.3b illustrates a core which is relatively flexible in shear, allowing the face sheets to slide with respect to one another. Figure 2.3c illustrates superposition of the two, representing the true behavior of a sandwich panel.

It is also extremely important that the adhesive used as a face sheet to core bonding mechanism should not be sufficiently flexible, which would allow substantial relative movements of the faces and the core.

2.3 PROPOSED 3-D GFRP SANDWICH PANELS

There are several concerns associated with the design of conventional sandwich panels which can be related to the various failure modes of sandwich panels as illustrated in Figure 2.4. The most common problem in FRP sandwich structures arises when the face sheets debond from the core material. The initiation of the damage can be caused by manufacturing imperfections or by impact forces applied to a sandwich panel. The second underlying problem concerning sandwich panels occurs when the core is very flexible in shear. Deflections now become a function of not only bending, but are also dependent on shear. In certain cases, the contribution of shear deformation to the total deflection can exceed that of bending (Allen, 1998). Polyurethane foam cores are
typically much weaker than equivalent density honeycomb counterparts. Table 2.1 provides some typical properties of various core materials. Also, weaker, less dense foam cores may allow sudden failures when the face sheets buckle on the compression side (Bitzer, 1997). The durability of metallic sandwich panels is especially vulnerable to corrosion of the face sheets which can eventually lead to bonding interface degradation. Furthermore, sandwich structures are notoriously sensitive to localized external lateral loads such as point loads, line loads, or concentrated surface loads of high intensity. This pronounced sensitivity is due to the inducing of significant local deflections of the loaded face into the core material, thus causing high local stress concentrations.

Until recently, delamination problems were resolved using improved adhesive related connection details. The growing interest in composites and their applications has lead to the development of 3-Dimensional pultruded GFRP sandwich panels. Through-thickness (3-D) fibers are inserted perpendicularly between the face sheets to overcome the shortcomings of conventional sandwich structures by minimizing delamination and degradation of stiffness over time (Kim et al. 1999). The through thickness fibers provide an increase in the shear stiffness of the core and delay localized buckling of the face sheets. Furthermore, the use of GFRP offers a variety of advantages compared to conventional sandwich materials including ease of handling, excellent corrosion resistance characteristics, and rapid installation on site. One concern however, is the vulnerability of GFRP and polyurethane foam to ultraviolet radiation (ACI, 1996). Unprotected exposure to UV rays may cause the outer skins and core to become brittle and possibly lead to premature failure.
The proposed 3-D pultruded sandwich panels presented in this study consist of a closed-cell polyurethane foam core sandwiched between two GFRP laminate layers which are connected by through-thickness fibers as shown in Figure 2.5. As previously stated, the through-thickness fibers prevent delamination of the composite plates from the core while increasing the core shear modulus. The 3-D fibers also mitigate the effects of localized load deformations in the core under the face sheets. The panels are fabricated using a pultrusion process as shown in Figure 2.6, where the through thickness fibers are injected through the panels before the thermoset resin has cured. To begin the pultrusion process, varying layers of stitched fabrics are pulled together to produce the desired top and bottom skin thicknesses. The urethane foam core is inserted between the top and bottom layers to form the sandwich panel and maintain spacing. 3-D fibers are then inserted through the entire panel and cut to the desired length. Finally, epoxy resin infuses throughout the glass fibers of the panel which is then pulled through a heated die forming the cured composite structural member. The width of the panels can vary from 6 in. to 8 ft. 6 in. while the thickness can vary from ½ to 4 in. The fiber insertion density can vary from 0 to 16 fibers per square inch (fipsi). Skin ply thickness can vary from 0.1 to 0.5 in. The use of GFRP face sheets produces a lightweight, corrosion-resistant panel that is relatively low in cost compared to more exotic aerospace carbon-composite constructions. The cost of these GFRP sandwich panels can be favorably compared to more conventional construction materials especially when the reduced costs of installation and public inconvenience are considered. The 3-D GFRP sandwich panels are used in a variety of applications including truck trailer and rail car walls, floors, and
roofs, pedestrian bridge decks, construction panels, industrial and military mats, and marine decks and bulkheads.

2.4 CURRENT KNOWLEDGE

Previous research conducted at North Carolina State University’s Constructed Facilities Laboratory showed that various parameters affect the behavior of the proposed 3-D GFRP sandwich panels. These include fiber insertion density, fiber insertion pattern and test direction, panel thickness, and GFRP laminate skin thickness (Reis, 2005). It was shown in previous studies at NCSU that the compressive strength of a panel is directly proportional to the amount of fiber insertions per square inch (fipsi), but independent of fiber insertion pattern. Decreasing the thickness of the sandwich panels increased the buckling load of the 3-D fibers and thus increased the compressive strength.

It was shown that the in-plane tensile strength of the laminate skins decreased as the amount of fipsi increased. Increasing the 3-D fiber density from 8 fipsi to 16 fipsi resulted in a 25% decrease of the tensile strength of the face sheets. However, increasing the fiber insertion density appeared to have a negligible effect on the elastic modulus of the face sheets. The stress-strain relationship of the tensile specimens was bi-linear in nature, resulting from the presence of fibers in the perpendicular direction. The reduction of initial stiffness was approximately 33 percent for all tensile coupons tested.

The shear stress-strain behavior of the core was linear up to the initiation of the first crack in the foam followed by a non-linear behavior with a reduction in stiffness up to failure. The significant reduction in stiffness is a result of diagonal tension cracks propagating throughout the core and the formation of plastic hinges at each end of the through thickness fibers. Increasing the through thickness fiber density from 8 to 16
fipsi, without changing the configuration, resulted in a 33 percent increase in shear stiffness. More importantly, the presence of a continuous wall of fibers affected the shear stiffness significantly. Changing the configuration of the 3-D fiber insertion pattern, for a constant 4 fipsi, from a regular array type to a continuous wall pattern resulted in a 450 percent increase of initial core stiffness. It was shown that thickness of the panels did not have a significant effect upon shear stiffness but did reduce the shear strength considerably. This research also demonstrated the importance of the foam core in providing lateral restraint of the through-thickness fibers and thus increasing the core shear stiffness (Reis et al. 2004).
Figure 2.1 Honeycomb Core Configurations

Figure 2.2 Illustration of a Typical Sandwich Element
Figure 2.3 Effect of Core Shear Rigidity

- Figure 2.3a Core Rigid in Shear
- Figure 2.3b Core Flexible in Shear
- Figure 2.3c Superposition of 2.3a & 2.3b
1. **Facing failure**
   Initial failure may occur in either compression or tension face. Caused by insufficient panel thickness, facing thickness or facing strength.

2. **Transverse shear failure**
   Caused by insufficient core shear strength or panel thickness.

3. **Flexural crushing of core**
   Caused by insufficient core flatwise compressive strength or excessive beam deflection.

4. **Local crushing of core**
   Caused by low core compression strength.

5. **General buckling**
   Caused by insufficient panel thickness or insufficient core shear rigidity.

6. **Shear crimping**
   Sometimes occurs following, and as a consequence of, general buckling. Caused by low core shear modulus or low adhesive shear strength.

7. **Face wrinkling**
   Facing buckles as a plate on an elastic foundation. It may buckle inward or outward, depending on relative strengths of core in compression and adhesive in flatwise tension.

8. **Intracell buckling (dimpling)**
   Applicable to cellular cores only. Occurs with very thin facings and large core cells. This effect may cause failure by propagating across adjacent cells, thus inducing face wrinkling.

---

Figure 2.4 Failure Modes of Sandwich Panels
Table 2.1 Comparison of Core Material Properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (pcf)</th>
<th>Strength (psi)</th>
<th>Modulus (ksi)</th>
<th>Strength (psi)</th>
<th>Modulus (ksi)</th>
</tr>
</thead>
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<td>Aluminum Honeycomb</td>
<td>3.1</td>
<td>300</td>
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<td>210</td>
<td>45</td>
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<td>Nomex Honeycomb</td>
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<td>325</td>
<td>20</td>
<td>175</td>
<td>6</td>
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<td>Fiberglass Honeycomb</td>
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<td>23</td>
<td>195</td>
<td>19</td>
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<td>Rohacell Foam</td>
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<td>10</td>
<td>114</td>
<td>3</td>
</tr>
<tr>
<td>Klegecell Foam</td>
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<td>69</td>
<td>2.7</td>
<td>51</td>
<td>1.1</td>
</tr>
<tr>
<td>Rigicell Foam</td>
<td>3.0</td>
<td>80</td>
<td>2.5</td>
<td>70</td>
<td>2.5</td>
</tr>
<tr>
<td>Divinycell Foam</td>
<td>3.1</td>
<td>100</td>
<td>10.2</td>
<td>73</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Figure 2.5 Innovative 3-D GFRP Sandwich Panel
Figure 2.6 Pultrusion Process
CHAPTER 3

3 EXPERIMENTAL PROGRAM

3.1 INTRODUCTION

This chapter summarizes the research program undertaken at North Carolina State University’s Constructed Facilities Laboratory to determine the fundamental characteristics of the proposed 3-D GFRP pultruded sandwich panels currently under development by a local manufacturer. The experimental program is designed to determine the in-plane tensile properties of the laminate face sheets, shear properties of the core, and flexural characteristics of the composite sandwich panels.

This research program primarily investigates the effect of various through thickness fiber insertion patterns, selected by the manufacturer, on the structural behavior of the sandwich panels. Each distinct pattern incorporates several detailed parameters such as test direction, intra-wall spacing, and inter-wall spacing. In this experimental program, the x-direction is always referring to the direction parallel to the pultrusion direction. The y-direction is the direction perpendicular to the x-direction, however, still in the plane of the face sheet. The 3-D insertion patterns considered in this program are of three main types: regular array, “continuous” wall pattern, and mat or “checkerboard” pattern. A graphic illustration of the various patterns is shown in Table 3.1. Patterns 1 and 2 are of the regular array type, in which the 3-D fibers are inserted at regular intervals which can vary in the x- and y- directions. The through thickness fibers in Pattern 1 are spaced at 0.333” in the x-direction while y-direction spacing is slightly larger at 0.375”. Pattern 2 has equal spacing of fibers in both x- and y- directions at 0.25”. Patterns 3 and
are of the “continuous” wall type, in which spacing in the y-direction is quite small relative to the spacing in the x-direction to simulate a “continuous” wall of fibers. Pattern 3 is divided into two parts, 3a and 3b. However, specimens for Pattern 3a were not provided and thus not considered in this study. Pattern 3b consists of 3-D fibers spaced at 0.15” in the y-direction (intra-wall) whereas the “continuous” walls are spaced at 1.0” apart in the x-direction (inter-wall). Pattern 4 has the same intra-wall spacing of 0.15” yet spacing of the walls is now 2.0” apart in the x-direction. Patterns 5 and 6 are referred to as “checkerboard” patterns where there are walls in both the x- and y-directions. Walls remain continuous in the y-direction whereas in the x-direction sets of shorter walls are offset in an alternating, checkerboard like pattern. The only difference between Patterns 5 and 6 is the spacing of the continuous walls in the x-direction. The inter-wall spacing in Pattern 5 is 1.5” compared to 1.95” for Pattern 6. One important parameter affecting the structural behavior is “fipsi” or fiber insertions per square inch. Variation of the x- and y-direction spacing has a direct effect on the amount of fipsi, which serves as an overall quantitative measure of 3-D fiber reinforcement. For instance, decreasing the spacing in either direction would result in an increase in the amount of fipsi. With these fundamental understandings of the types of sandwich panels considered in this program, the experimental research program is discussed in the following section.

3.2 TENSION TESTS

The main objective of the experimental tension program was to evaluate the in-plane tensile characteristics of the 4-ply GFRP stitched laminate face sheets and determine the effects, if any, of the through-thickness fiber insertion pattern and the
direction of the test. These results provided essential information for an accurate analysis of the flexural behavior. A total of 18 tensile coupons, including three replicates for each of Patterns 4, 5, and 6 in both x- and y-directions, were tested in accordance with ASTM D3039 (2006). Pattern 4 is of the “continuous” wall type while Patterns 5 and 6 are of the mat or “checkerboard” type. The specimens were cut from untested flexural samples to determine the effect, if any, of the through thickness fiber configuration on tensile strength and stiffness. Tensile samples of Patterns 4, 5, 6 oriented in both x- and y-directions are shown in Figure 3.1. A nomenclature for the tensile specimens was developed as follows: T-(RDPS4-5)-x-2 where T represents Tension, RDPS4 refers to Research and Development Panel Set 4, 5 indicates Fiber Insertion Pattern 5, x corresponds to testing in the x-direction, and 2 refers to specimen number 2.

3.2.1 TENSION TEST SETUP

The tension coupon specimens were cut from the face sheets of untested 3-D GFRP sandwich panels. The overall dimensions are 3 in. wide and 18 in. long with an average thickness of 0.16 in. The samples were cut using a wet saw with a diamond tipped blade and were then sanded to ensure consistency of the cross-sectional area. Four aluminum tabs, 5.5 in. long, were bonded to each side at each end of the samples using Sikadur 32, Hi-Mod structural adhesive to prevent crushing and localized stress concentrations under the 3,000 lbs of gripping force, as shown in Figure 3.2. One Vishay C2A-13-250LW-350 series, 350 Ω electric resistance foil strain gage with a gage length of 0.25 in., was attached at the midspan to the exterior, smooth side of the coupons as shown in Figure 3.3. The other side of the coupon specimens, although sanded for area
consistency, was too rough due to core fabrication for a proper strain gage installation. A three lead wire 350 Ω gage subjected to 2V of excitation was used as opposed to a 120 Ω gage because FRP materials do not dissipate heat well and thermal expansion effects of a foil gage could affect the results. Since a strain gage could be applied to only one side, extraordinary care was taken during gripping to avoid any major eccentricity. Monotonic loading was applied using a ± 200 kip MTS hydraulic actuator at a rate of 0.025 in/min until fiber rupture occurred. Load values were obtained from the internal load cell of the MTS machine. Data was acquired from the instrumentation using a Vishay Model 5100b scanner and Vishay Strain Smart v. 4.01 data acquisition software.

3.3 SHEAR TESTS

The shear behavior considered in this research program was found to be an important aspect affecting the overall behavior of the proposed 3-D sandwich panels. This was due to the inherent characteristic of sandwich panels to exhibit a large influence of shear on the overall structural behavior. The main objective of this phase was to determine the influence of fiber insertion pattern and test direction on the core behavior of the innovative 3-D GFRP sandwich panels. A total of 50 specimens, 4 replicates for each varying through-thickness fiber insertion pattern and test direction (Pattern - Test Direction) 1-x, 2-x, 2-y, 3b-x, 3b-y†, 4-x, 4-y†, 5-x, 5-y, and 6-x, 6-y were tested in accordance to ASTM C273 (2000). A detailed test matrix of the experimentally evaluated shear specimens can be found in Table 3.2. It should be noted that the length to thickness ratio does not match the ASTM C273 recommendation of 12:1. However, ASTM C273 does permit variation as long as the purchaser and seller are in agreement.
The lower length to depth ratio of 6:1, which was typical of these shear specimens, could have an effect on the stress distribution in comparison to thinner sandwich panels. Consequently, the measured shear strengths reported in this study may be less than the actual shear strengths of the sandwich panels. However, the selected dimensions of the specimens tested in this program were remaining consistent with previous research conducted on similar panels at NCSU. A nomenclature for the shear specimens was developed as follows: S-(RDPS4-5)-2.0-5-4-y-3 where S represents Shear, RDPS4 refers to Research and Development Panel Set 4, 5 indicates Fiber Insertion Pattern 5, 2.0 delegates a 2.0 in. overall thickness, 4 refers to the number of plies in the face sheets, y corresponds to testing in the y-direction, and 3 refers to specimen number 3. A graphical representation of the shear specimen nomenclature is shown in Figure 3.4.

† - 3 repeat tests for 3b-y and 4-y fiber insertion configurations were conducted to investigate the effect of through thickness fiber resin content

3.3.1 SHEAR TEST SETUP

The typical overall dimensions for the shear specimens, as shown in Figure 3.5, were 4 in. wide, 11.5 in. long, and 2.0 in. sandwich thickness. The outer face sheets of the specimens were sanded and cleaned with acetone to ensure adequate bonding to the steel plates. The specimens were then bonded using Sikadur 32, Hi-Mod structural epoxy to two 0.75 in. thick steel plates, one on each side. The two part epoxy compound was allowed to cure for at least 72 hours to ensure that adequate bond and shear strengths were reached. The bonded shear specimens were then placed in the testing apparatus as
shown in Figure 3.5, which was designed to have the load line of action passing through the diagonally opposite corners. This testing configuration was accomplished through the use of the angled cut, bonded steel plates and v-notched plates at the top and bottom as shown in Figure 3.6. Monotonic compressive loading was applied through a stroke controlled rate of 0.02 in/min until a displacement of approximately 0.75 in. was reached. This level was the limit to which the steel plates could move without making contact with other components of the test setup. The relative deflection between the two plates was measured using two 1.0 in. potentiometers placed at midspan on opposite sides. Load was obtained using a calibrated 25 kip load cell. The data from the instrumentation was acquired using a Vishay Model 5100b scanner and Vishay Strain Smart v. 4.01 data acquisition software.

3.4 FLEXURE TESTS

The main objective of this phase of the experimental program is to determine the effect of various configurations of through-thickness fiber insertions, panel widths, and span lengths on the flexural behavior of the proposed 3-D GFRP sandwich panels. This research phase is essential to describe the overall behavior and provide evaluation for the analytical modeling. A total of 138 flexural specimens, 2.0 in. thick with varying widths of 3.0 and 6.0 in. and span lengths of 28, 40, 52, and 64 in. were tested in accordance with ASTM C-393 (2000). The through thickness fiber insertion patterns and test directions evaluated in this portion were restricted to (Pattern - Test Direction) 4-x, 4-y, 5-x, 5-y, and 6-x, 6-y due to availability of test specimens. A detailed test matrix of the experimentally evaluated flexural specimens can be found in Table 3.3. A nomenclature
for the flexure specimens was developed as follows: F-(RDPS4-5)-2.0-5-4-y-52-6-3
where F represents Flexure, RDPS4 refers to Research and Development Panel Set 4, 5
discerns Fiber Insertion Pattern 5, 2.0 delegates a 2.0 in. overall thickness, 5 again
discerns Fiber Insertion Pattern 5, 4 refers to the number of plies in the face sheets, y
corresponds to testing in the y-direction, 52 indicates a span length of 52”, 6 implies a
width of 6.0”, and 3 refers to specimen number 3. A graphical representation of the
flexure specimen nomenclature is shown in Figure 3.7.

3.4.1 FLEXURE TEST SETUP
All flexural specimens tested had the same overall thickness of 2.0 in., but the
widths varied from 3.0 in. to 6.0 in. and span lengths varied from 28 in., 40 in., 52 in., to
64 in. Span length values correspond to the length of the sample from support to support.
The flexural specimens had a 2.0 in. overhang beyond each end of the supports.
Specimens were simply supported, and tested in 3-point bending with a concentrated
point load applied through a rounded knife-edge support at the midspan as shown in
Figure 3.8. Three replicate specimens were evaluated for each configuration with one
specimen having a Vishay C2A-13-250LW-350 series, 350 Ω electric resistance strain
gage applied to the tension face sheet at the midspan, as shown in Figure 3.9. Monotonic
loading was applied using a ± 200 kip MTS hydraulic actuator at a stroke rate of 0.5
in/min until a level of approximately 5.0 in. was reached. This level was the limit to
which the specimen could deform without making contact with other components of the
test setup. The midspan deflection of the sample was measured by a 12 in. string
potentiometer placed in between the double channel assembly, as shown in Figure 3.10.
The potentiometer was attached to a screw eye in a small wooden block which was epoxied to the midspan of the specimen on the tension side, which is also shown in Figure 3.10. Load values were measured using the same calibrated 25 kip load cell. The data from the instrumentation was acquired using a Vishay Model 5100b scanner and Vishay Strain Smart v. 4.01 data acquisition software.
Table 3.1 Comparison of 3-D Fiber Insertion Patterns

<table>
<thead>
<tr>
<th>Core Pattern</th>
<th>x-direction spacing</th>
<th>y-direction spacing</th>
<th>fipsi</th>
<th>3-D Fiber Configuration</th>
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<tr>
<td>1</td>
<td>0.333</td>
<td>0.375</td>
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<td>2</td>
<td>0.25</td>
<td>0.25</td>
<td>8.0</td>
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<tr>
<td>3b</td>
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<td>0.15</td>
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<tr>
<td>4</td>
<td>2.0</td>
<td>0.15</td>
<td>6.67</td>
<td><img src="Diagram4.png" alt="Diagram 4" /></td>
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<tr>
<td>5</td>
<td>1.5</td>
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<td>8.53</td>
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<td>1.95</td>
<td>0.15</td>
<td>7.59</td>
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Figure 3.1 Tension Coupon Samples

Figure 3.2 Tension Coupon with Bonded Aluminum Tabs
Figure 3.3 Strain Gage Bonded to Tension Coupon
Table 3.2 Shear Specimen Test Matrix

<table>
<thead>
<tr>
<th>Core Pattern</th>
<th>Shape</th>
<th>Test Direction</th>
<th>Specimen Designation</th>
<th># of Specimens</th>
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<tr>
<td>1</td>
<td></td>
<td>x</td>
<td>S-(RDPS4-1)-2.0-1-4-x-#</td>
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<tr>
<td></td>
<td></td>
<td>y</td>
<td>S-(RDPS4-1)-2.0-1-4-y-#</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>x</td>
<td>S-(RDPS4-2)-2.0-2-4-x-#</td>
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<tr>
<td></td>
<td></td>
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<td>3b</td>
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<td></td>
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<tr>
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<td></td>
<td></td>
<td>y</td>
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<td>4</td>
</tr>
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† - 3 repeat tests for 3b-y and 4-y fiber insertion configurations were conducted to investigate the effect of through thickness fiber resin content
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<th>Specimen</th>
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<td>1</td>
</tr>
<tr>
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<td>4</td>
<td>Y</td>
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<td>3b</td>
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Figure 3.4 Shear Specimen Nomenclature

![Shear Test Setup](image)

Figure 3.5 Shear Test Setup
Figure 3.6 V-notched Steel Plates
Table 3.3 Flexure Specimen Test Matrix

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</table>

Note: £ field in specimen designation represents all span lengths of 28, 40, 52, and 64 in.

*One less specimen of 64 in. span length due to use in fabrication of tension samples
Figure 3.7 Flexure Specimen Nomenclature

F- (Name) - Thickness - Pattern - Plies - Direction - Span - Width - Specimen

F- (Name) - a - b - c - d - e - f - #

1.0 1 2 X 28 3 1
1.5 2 4 Y 40 6 2
2.0 3a 6 52 3
4.0 3b 6 64
4
5
6

Figure 3.8 Flexure Test Setup
Figure 3.9 Strain Gage Bonded to Midspan of Flexural Specimen

Figure 3.10 String Potentiometer Location
CHAPTER 4

4 TEST RESULTS & DISCUSSION

4.1 INTRODUCTION
This chapter presents and discusses test results of the experimental program undertaken to determine the material characteristics of the proposed 3-D GFRP pultruded sandwich panels. The experimental program was designed to determine the in-plane tensile properties of the laminate face sheets, shear properties of the core, and the flexural behavior of the composite sandwich panels. Based on experience gained in this program, other issues associated with the manufacturing process of the proposed sandwich panels are also discussed.

4.2 TENSION TESTS RESULTS AND DISCUSSION

In reference to the test setup described in Section 3.3, the axial normal stress, \( \sigma \), was determined based on the measured applied load \( P \) as follows:

\[
\sigma = \frac{P}{A}
\]

where \( A \) is the cross sectional area of the tension coupon. The normal strain, \( \varepsilon \), was measured directly using a 350 \( \Omega \) uniaxial electric resistance strain gage. Based on the above, the stress-strain relationship was established for all the tested tension specimens, as typically shown in Figure 4.1. The bilinear stress-strain relationship shown in the figure is most likely due to the effect of the through thickness fibers perpendicular to the direction of the fibers in tension. Failure of the specimens occurred due to rupture of the glass fibers at an approximate strain of 1.0 to 1.5 percent. The measured rupture strains
were consistent with the measured failure strains reported previously for E-glass laminate face sheets of similar sandwich panels (Reis 2005). The initial elastic modulus, $E_1$, was determined based on the slope of the stress-strain relationship within a strain range of 0.05 percent, to account for grip seating, up to a strain of 0.3 percent. The second elastic modulus, $E_2$, was determined within a strain range of 0.3 percent to 0.8 and in some cases up to 1.1 percent. The second strain range value was determined by visual inspection of the linearity of the stress-strain relationship. The initial elastic modulus was on average 25 percent higher than the second elastic modulus.

It was observed that neither the core pattern, test direction, nor the amount of fipsi had a discernable effect upon the elastic moduli. However, test results indicate that the test direction and fiber insertion pattern have an effect upon the fiber rupture failure modes and ultimate strength. Figure 4.2 illustrates the typical failure mode of a tested tension specimen with a “continuous” wall through-thickness fiber configuration, tested in the direction perpendicular to the walls. Failure occurred due to fiber rupture along the 3-D fiber insertion pattern. Figure 4.3 illustrates the typical failure mode of a tension specimen with a “checkerboard” through-thickness fiber configuration, tested in the direction perpendicular to the continuous walls. Consistent with the “continuous” wall pattern, failure of the “checkerboard” pattern occurred due to fiber rupture along the 3-D fiber insertions. These results suggest that the through-thickness fiber insertions create zones of imperfection, and discontinuities, which initiate failure at these locations.

The ultimate strength of a tensile specimen is also dependent upon the configuration and test direction of the 3-D fiber insertions. For “continuous” wall patterns, the direction perpendicular to the wall insertions failed at an average ultimate
strength of 30.9 ksi compared to an ultimate strength of 37.2 ksi the direction parallel to the continuous wall insertions. However, for “checkerboard” through-thickness fiber configurations, testing in the direction perpendicular to the continuous wall insertions resulted in higher ultimate strength values. “Checkerboard” fiber insertion pattern 5, tested in the direction perpendicular to the continuous walls, had an ultimate strength of 34.6 ksi compared to an ultimate strength of 30.4 ksi in the continuous wall direction. “Checkerboard” insertion pattern 6, tested in the direction perpendicular to the continuous walls, had an ultimate strength of 35.1 ksi compared to an ultimate strength of 28.5 ksi in the continuous wall direction. The results of these tests again indicate the effect of discontinuities created by the through-thickness fiber insertions. The stress-strain relationships of all tested tension specimens are given in Appendix A.

4.3 SHEAR TESTS RESULTS AND DISCUSSION

Based on the test setup described in Section 3.4, the shear stress, \( \tau \), should be determined using the applied load \( P \), Length, \( L \), and Width, \( w \), of the specimen as follows:

\[
\tau = \frac{P}{(L \cdot w) \cos(\alpha)}
\]  
(4.2)

where \( \alpha \) is the angle of inclination relative to the vertical axis. Since the 2.0 in. thick shear specimens involved in this research program resulted in an angle of inclination which was less than 10 degrees, a small angle approximation was assumed. Thus, the shear stress, \( \tau \), was calculated as:

\[
\tau = \frac{P}{(L \cdot w)}
\]  
(4.3)
Engineering shear strain, $\gamma$, was also calculated using a small angle approximation formula:

$$\gamma = \frac{\Delta}{c}$$

(4.4)

where $\Delta$ is the average relative displacement measured by the potentiometers and $c$ is the sandwich panel core thickness. All shear test results indicate a somewhat linear shear stress-strain relationship up to the initiation of the first crack of the foam core followed by a non-linear behavior with significant reduction in shear stiffness, as shown in Figure 4.4 for a regular array fiber insertion pattern. The reduction in stiffness can be attributed to diagonal tension cracks propagating throughout the polyurethane foam core, as shown in Figure 4.5, and the formation of plastic hinges at both ends of the through-thickness fibers. The initial shear modulus, $G_1$, was determined based on the slope of the stress-strain curve within a range of 5 psi shear stress, to account for test setup seating and potentiometer sensitivity range, up to 0.01 radians shear strain. The second shear modulus, $G_2$, was determined within the shear strain range of 0.01 radians to 0.03 radians. The strain range values were determined based on the experiences gained in this study and previous research investigations. A graphic representation of the initial and second shear modulus ranges is shown in Figure 4.6.

Experimental results indicate that the 3-D fiber insertion configuration and test direction significantly affect the shear behavior of the 3-D sandwich panels. Based on a thorough inspection of the behavior of the tested shear specimens, it became obvious that the overall shear behavior should be divided into two main categories: non-wall shear and wall-shear. Non-wall shear applies to regular array patterns tested in both directions, as shown in Figure 4.7, and “continuous” wall patterns tested only in the direction
perpendicular to the wall insertions, as shown in Figure 4.8. As illustrated in the figures, the shear stress-strain response of these different 3-D fiber configurations is similar in nature. Wall-shear is used to characterize “continuous” wall patterns tested in the direction parallel to the wall insertions, as shown in Figure 4.9, and “checkerboard” patterns tested in both directions, as shown in Figure 4.10 and Figure 4.11. The presence of a “continuous” wall of through-thickness fibers creates a mechanism similar to that of a shear wall, thus minimizing the stress concentrations at the joint connections of 3-D fibers and increasing the shear modulus of the sandwich panel considerably. However, the reduction in initial stiffness is significantly greater for wall-shear specimens when compared to that of non-wall shear specimens, as shown in Table 4.1. Thus, wall-shear panels have a higher degree of nonlinearity in the shear stress-strain relationship where non-wall shear panels remain approximately linear in the strain range up to 0.03 radians, as shown in Figure 4.12.

For regular array patterns, decreasing the spacing of the through-thickness fibers increases the shear stiffness in approximately a linear relationship.

The response of “continuous” wall patterns tested in the direction parallel to the wall insertions as opposed to perpendicular to the wall insertions resulted in roughly a 200 percent increase in the shear stiffness of the specimens. Decreasing the inter-wall spacing from 2.0 in. to 1.0 in. resulted in a 24 percent increase in the initial stiffness from 1.13 ksi to 1.40 ksi, when tested in the direction parallel to the wall insertions. Decreasing the inter-wall spacing from 2.0 in. to 1.0 in. had no apparent affect upon initial stiffness, when tested in the direction perpendicular to the wall insertions.
However, average final stiffness values in the perpendicular wall insertion direction increased by 12.5 percent from 0.40 ksi to 0.45 ksi.

“Checkerboard” 3-D fiber insertion patterns, consisting of staggered wall insertions perpendicular to the continuous wall insertions, resulted in the highest shear stiffness values. The shear stiffness results were similar in each perpendicular direction due to the presence of walls in the both directions. Although inter-wall spacing of the staggered walls remained at a constant value of 0.75 in., variation of the continuous wall spacing was examined. Test results showed that decreasing the continuous inter-wall spacing from 1.95 in. to 1.50 in., as in the case of changing from “checkerboard” pattern 6 to pattern 5, resulted in an increase of the initial stiffness in both directions. “Checkerboard” pattern 5, having the highest overall fipsi of 8.53, was the strongest pattern tested with $G_1$ values in both directions on the range of 2.60 ksi.

For wall-shear panels, initially, the shear stress-strain relationship remains approximately linear up to the initiation of diagonal tension cracks in the polyurethane foam core and minor joint deterioration at both ends of the 3-D fibers, as shown in Figure 4.13. A significant reduction in stiffness is observed as the foam cracking propagates and progressive joint deterioration continues. Once the joints at both ends of the through-thickness fibers have essentially become plastic hinges, membrane forces develop and the fibers act as free links resisting tensile forces. Since fibers are well suited for resisting axial tension, at high levels of shear strain of approximately 0.25 radians, the stiffness of the sandwich panel started to increase. Figure 4.13 provides a graphic illustration of this progressive failure behavior and annotates with the suggested explanation.
In addition, two specimens of the “continuous” wall pattern 4 tested in the direction perpendicular to the wall insertions, and one specimen of the “checkerboard” pattern 6 tested in the direction parallel to the continuous wall insertions, were subjected to cyclic loading. The loading was applied in multiplicative intervals of 0.25, 0.5, 0.75, 1.0, 2.0, 3.0, and 4.0 times the “yield” load or value at which the previous monotonically tested specimens started to show a nonlinear response. The cyclic shear stress versus shear strain results of these tests is shown in Figures 4.14, 4.15, and 4.16 respectively. These figures indicate that as shear stress levels increase there is a corresponding increase in residual shear strain. Figure 4.17 compares the percentage of maximum shear stress attained versus the amount of residual shear strain for one “continuous” wall specimen, S-(RDPS4-4)-2.0-4-4-x-3, tested in the direction perpendicular to the wall insertions. At values of roughly 20 and 48 percent of the maximum shear stress, 0.01 rad and 0.03 rad shear strain levels were respectively reached, indicated by the vertical lines. Remaining below roughly 65 percent of the maximum shear stress during loading conditions will ensure that once unloaded, a panel will remain in the initial elastic portion up to 0.01 rad shear strain. For overloading conditions, remaining below roughly 80 percent of the maximum shear stress will ensure a panel will remain in the second elastic portion up to 0.03 rad shear strain. Figure 4.18 and Figure 4.19 illustrate similar comparisons for “continuous” wall specimen S-(RDPS4-4)-2.0-4-4-x-4 and “checkerboard” fiber insertion specimen S-(RDPS4-6)-2.0-6-4-y-4 respectively. The stress-strain relationships of all tested shear specimens are given in Appendix A.
4.4 FLEXURE TESTS RESULTS AND DISCUSSION

The typical measured load versus midspan deflection relationship of the tested flexure specimens was linear up to the initiation of the first crack in the foam core followed by a non-linear behavior, as shown in Figure 4.20. The majority of the failures (94 percent) observed for the tested flexural sandwich specimens could be characterized as excessive midspan deflection, as shown in Figure 4.21. The remaining (6 percent) failed due to crushing of the fibers in the compression zone, as shown in Figure 4.22. A list of the tested specimens which failed by compression zone fiber crushing can be found in Table 4.2. All of the specimens that failed due to crushing of the fibers in the compression zone were of “checkerboard” insertion patterns 5 and 6 and of shorter span lengths 28 in. and 40 in. Crushing of the fibers in the compression zone normally occurred directly under the point of load application. In one case however, the crushing occurred at an offset distance several inches from the point of loading. Further inspection revealed that the crushing of the fibers in the compression zone occurred adjacent to 3-D fiber insertions, more specifically wall insertions, as shown in Figures 4.23, 4.24, and 4.25. The observed failure mechanism suggests that the through-thickness fiber insertions create a zone of imperfection or “waviness” in the laminate skins, similar to what was experienced and discussed in the tension specimens. This behavior was evident by the fact that all of the premature failure specimens were of “checkerboard” patterns 5 and 6 which have the highest amount of fiber insertions per square inch, thus the highest amount of distortion.

As expected, for a certain core pattern configuration and width, the specimens with a shorter span length reached higher level loads at a given deflection, as shown in Figure 4.26. Test results indicate the shear characteristics of the core have a dominate
influence upon the flexural behavior. Thus “continuous” wall patterns tested in the direction parallel to the wall insertions were stiffer when compared to the direction perpendicular to the wall insertions. Overall, the “checkerboard” patterns resulted in the highest flexural stiffness for any through-thickness fiber configuration. Comparison of specimens with a 3.0 in. width versus 6.0 in. width, are shown for “checkerboard” pattern 5 tested in the direction perpendicular to the continuous wall insertions in Figure 4.27. The behavior indicates that the width is not an important flexural parameter since doubling the width resulted in twice the amount of load for a given deflection. Thus, engineers could consider the load per unit width in the design guidelines. Load versus midspan deflection relationships of all tested flexure specimens are given in Appendix A.

4.5 MANUFACTURING RELATED ISSUES DISCUSSION

Several manufacturing issues have been identified throughout the testing program conducted on the proposed 3-D pultruded sandwich panels. The following section discusses issues related to skewed orientation of the fibers in the GFRP laminate skins, resin content of the 3-D fiber insertions, and nonconforming through thickness fiber insertions.

4.5.1 GFRP LAMINATE SKINS

It was noticed that for many samples, the fiber orientation of the panel face sheets is typically not aligned with the pultrusion or through-thickness fiber insertion direction as shown in Figure 4.28. The stitched, 0-90° E-glass fabric skin material, produced by Vectorply, has a tendency to deform, allowing rotation of the fibers and thus not preserving the perpendicularity of the laminate layers. This provides a means for axial-
shear coupling which in turn results in warping and lowers ultimate strength. Based on examination of the fabrication process of these panels it was observed that the stitched fiber may be slightly skewed before ever being incorporated into the manufacturing of the 3-D panels. This is most likely due to the low amount of restraint provided by the stitching. A woven fabric may be more suitable to diminish this problem; however, it would most likely result in an increased cost. Another, highly probable cause for the fiber angle skew is the amount of friction between the epoxy saturated skins and the heated die on the pultrusion machine.

4.5.2 FIBER RESIN CONTENT

It was hypothesized that uniform resin absorption by the face sheets, through-thickness fibers, and core would be guaranteed. Referring to shear tests S-(RDPS4-3)-2.0-3b-4-y-# and S-(RDPS4-4)-2.0-4-4-y-#, shown in Figure 4.29, it was observed that decreasing the spacing of the “continuous” walls from 2.0” to 1.0”, as is the case in going from core pattern 4 to 3b, resulted in an unexpected decrease of $G_1$ from 1.87 ksi to 1.31 ksi and $G_2$ from 0.82 ksi to 0.55 ksi. These measured values are quite contradictory to previously obtained results from an experimental investigation on similar panels. In an effort to inspect the 3-D fibers, the polyurethane foam core was first treated with a solvent N-Methyl-Pyrrolidinone, commonly used in industrial urethane applications. The core was allowed to dry for 24 hours and then sandblasted with a mixture of silica sand and foam particulates so as not to damage the through-thickness fibers. Upon examination, it was clear there was more resin surrounding the through-thickness fibers in core pattern 4, thus creating a closer spaced “continuous” shear wall, as shown in Figure 4.30. In order to verify the effect of the finding, additional shear samples for both
patterns 3b and 4 were requested from Martin Marietta Composites. Three samples of each were tested in the direction parallel to the continuous walls, designated by S-(RDPS4-3)-2.0-3b-4-y-## and S-(RDPS4-4)-2.0-4-4-y-##. The results as shown in Figure 4.31 were in direct contradiction with the previously attained data. In these tests, decreasing the spacing of the “continuous” walls from 2.0 in. to 1.0 in. as is the case in going from core pattern 4 to 3b, resulted as expected, in an increase of G$_1$ from 1.13 ksi to 1.50 ksi and G$_2$ from 0.48 ksi to 0.65 ksi. Inspection of the through-thickness fibers in these samples revealed relatively equal amounts of resin content surrounding the fibers, as shown in Figure 4.32. The initial and subsequent specimens for Pattern 3b appeared to have equal resin saturation contents; therefore the results from the two sets of tests were averaged. Since the initial specimens for core Pattern 4 had a large amount of resin content surrounding the through-thickness fibers and the results were inconsistent with theory and previous work, the results from the series of latter tests were used. A quantitative shear modulus comparison of the initial test results to the latter series of test results is given in Table 4.3.

4.5.3 THROUGH-THICKNESS FIBER INSERTIONS

In theory, the 3-D fiber insertions are assumed to remain straight throughout the thickness of the sandwich panel. However, an inherent waviness in most of the fibers was observed, as shown in Figure 4.33. This geometric inconsistency is the result of the pultrusion manufacturing process when the 3-D fibers are injected through the wetted panel skins before the thermoset resin has cured. Once the fibers have been injected they are cut and due to the added resin weight and insufficient restraint, some sagging occurs. Although this investigation did not evaluate the differences between straight and
misaligned fibers, it is theoretically consistent that the wavy fibers would be more susceptible to buckling from induced compression forces.

On several occasions, a few non-conforming 3-D fiber insertions were noticed in flexural samples. Figure 4.34 illustrates a double wall through thickness fiber insertion in a sample which should only have regularly spaced single wall insertions. Figure 4.35 shows a diagonal through-thickness fiber insertion, and although it may be a desirable reinforcement detail, it was inconsistent with the core pattern being tested. Flaws in the 3-D fiber insertions were only noticed in several specimens of our panel set. For a large volume production plant, however, this could be a quality control issue worth further investigation.
Figure 4.1 Typical Stress-Strain Relationship of Tension Coupon
Figure 4.2 Fiber Rupture Mode of Tension Coupon – Core Pattern 4-x

Figure 4.3 Fiber Rupture Mode of Tension Coupon – Core Pattern 5-x
Figure 4.4 Typical Shear Stiffness Reduction

Figure 4.5 Diagonal Tension Cracking in Foam Core
Figure 4.6 Initial ($G_1$) and Final ($G_2$) Shear Modulus Ranges

Figure 4.7 Typical Non-Wall Shear Stress-Strain Relationship of Regular Array ($x,y$)
Figure 4.8 Typical Non-Wall Shear Stress-Strain Relationship of Continuous Wall (x)

Figure 4.9 Typical Wall Shear Stress-Strain Relationship of Continuous Wall (y)
Figure 4.10 Typical Wall Shear Stress-Strain Relationship of Checkerboard Pattern (x)

Figure 4.11 Typical Wall Shear Stress-Strain Relationship of Checkerboard Pattern (y)
Table 4.1 Material Properties of Tested 3-D GFRP Specimens

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Progressive deterioration of joints at 3-D fiber/skin interface

1st visible cracking of foam followed by epoxy cracking at rigid 3-D fiber joints

3-D fibers now acting as “free links” carry tensile forces and stiffness increases

Figure 4.12 Non-Wall Shear vs. Wall Shear Linearity

Figure 4.13 Proposed Wall Shear Specimen Progressive Failure Mechanism
Figure 4.14 Cyclic Shear Stress-Strain Results for Specimen S-(RDPS4-4)-2.0-4-4-x-3

Figure 4.15 Cyclic Shear Stress-Strain Results for Specimen S-(RDPS4-4)-2.0-4-4-x-4
Figure 4.16 Cyclic Shear Stress-Strain Results for Specimen S-(RDPS4-6)-2.0-6-4-y-4

Figure 4.17 Percent Maximum Shear Stress vs. Residual Shear Strain Comparison for Specimen S-(RDPS4-4)-2.0-4-4-x-3
Figure 4.18 Percent Maximum Shear Stress vs. Residual Shear Strain Comparison for Specimen S-(RDPS4-4)-2.0-4-4-x-4

Figure 4.19 Percent Maximum Shear Stress vs. Residual Shear Strain Comparison for Specimen S-(RDPS4-6)-2.0-6-4-y-4
Figure 4.20 Typical Flexure Load versus Midspan Displacement Relationship

Figure 4.21 Excessive Deflection Failure Mode
Figure 4.22 Crushing of the Fibers in the Compression Zone
Table 4.2 Flexure Specimens which Failed Prematurely due to Compression Fiber Rupture

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<td>X</td>
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<td>Under Point Load</td>
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<table>
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<th>y-spacing</th>
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<th>G_{2x} (ksi)</th>
<th>G_{1y} (ksi)</th>
<th>G_{2y} (ksi)</th>
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</table>
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5 ANALYSIS & CONCLUSIONS

5.1 INTRODUCTION
Analytical modeling of the innovative 3-D GFRP sandwich panels has been performed using two theories: Elementary Sandwich Theory (EST) and Advanced Sandwich Theory (AST). Both theories were proposed by Allen in 1969 and republished in 1998. Elementary Sandwich Theory is similar to ordinary engineering beam theory with the addition of the shear deformation term and is often referred to as “Timoshenko Beam Theory”. Advanced Sandwich Theory considers both bending and shear deformation, but in addition realizes the faces must bend locally in order to follow the shear deformation of the core, and thus the additional shear deflections are reduced by the local bending stiffness of the faces. Higher order analytical models have been developed by Frostig et al. (1991) to accommodate the cases when foam or low strength honeycomb cores change in height due to their transverse flexibility under loading conditions. However, these higher order analytical models are particularly computationally intensive for minimal gain in prediction accuracy; therefore they were neglected in this research study.

5.2 ELEMENTARY SANDWICH THEORY
The cross-section of a sandwich beam element, shown in Figure 5.1, is comprised of two thin face sheets of thickness t, separated by a thick, weaker core of thickness c. The overall depth of the beam is denoted by h while the distance between center lines of
the face sheets is considered to be the depth, \(d\), and \(b\) is the panel width. The flexural rigidity of the composite sandwich element can be determined as the sum of the individual flexural rigidities of the two separate face sheets and the core about the overall neutral axis of the section. Since the section is typically symmetric, the neutral axis lies directly at mid-height and the total flexural stiffness can be determined as:

\[
D = E_f \cdot \frac{bt^3}{6} + E_f \cdot \frac{btd^2}{2} + E_c \cdot \frac{bc^3}{12}
\]  

(5.1)

where \(E_f\) and \(E_c\) are the elastic moduli of the face sheets and core respectively. For typical sandwich construction, the second term is invariably dominant and the first term amounts to less than 1% of the second when:

\[
3 \left( \frac{d}{t} \right)^2 > 100
\]  

(5.2)

The third term also amounts to less than 1% of the second, and may be consequently neglected, when:

\[
6 \frac{E_f}{E_c} \frac{t}{c} \left( \frac{d}{c} \right)^2 > 100
\]  

(5.3)

For this research, \(E_c\) was an unknown value but was assumed to be negligible \((\approx 0)\) when compared to \(E_f\). This assumption prompted the consideration of an “antiplane” core here. Allen describes an antiplane core as, “an idealized core in which the modulus of elasticity in planes parallel with the faces is zero but the shear modulus in planes perpendicular to the faces is finite.” Using this definition and the assumption of, \(E_c = 0\), the antiplane core makes no contribution to the flexural stiffness of the beam. However, to remain as accurate as possible in regards to the calculations of normal stresses and deflections, the
first term was not neglected and the total flexural stiffness was calculated as a summation of the individual components of the face sheets.

5.2.1 BENDING & SHEAR STRESSES

The stresses in the face sheets and core are derived from Bernoulli-Euler Beam Theory with adaptation to the composite nature of the cross-section. Normal stresses in the outermost fibers of the face sheets can be calculated by the fundamental mechanics equation:

\[ \sigma = - \frac{Mz}{I} \]  \hspace{1cm} (5.4)

where \( M \) is the bending moment induced by loading, \( z \) is the distance from the neutral axis to the fiber under consideration, and \( I \) the moment of inertia about the neutral axis is calculated as:

\[ I = \frac{bt^3}{6} + \frac{btd^2}{2} \]  \hspace{1cm} (5.5)

For a simply supported beam subjected to a concentrated point load \( P \) at midspan, equation (5.4) simplifies to:

\[ \sigma = - \frac{PLh}{8I} \]  \hspace{1cm} (5.6)

The assumptions of ordinary beam theory also lead to the common expression for shear stress:

\[ \tau = \frac{VQ}{Ib} \]  \hspace{1cm} (5.7)

where \( V \) is the shear force induced by loading, \( I \) is the moment of inertia of the entire cross section about the neutral axis, \( b \) is the width of the section at a distance from the neutral axis, and \( Q \) is the first moment of area of the part of the section under
consideration. For a composite beam such as the sandwich element shown in Figure 5.1, the equation for shear stress must be modified to account for the various moduli of elasticity of the different elements of the cross section:

$$\tau = \frac{V}{Db} \sum QE$$  \hspace{1cm} (5.8)

In this equation, $D$ is the flexural rigidity of the entire section and $\sum QE$ represents the sum of the products of $Q$ and $E$ of all the components of the section. This expression results in the typical parabolic distribution of shear stress with the maximum value residing at the neutral axis. However, using the “antiplane” core assumption, where $E_c = 0$, the equation for shear stress in the core reduces to:

$$\tau = \frac{V E_t td}{D \cdot 2}$$  \hspace{1cm} (5.9)

with a constant shear stress distribution in the core and parabolic distribution in the face sheets. Here, since we are focusing on the core shear response, neglecting the flexural rigidity of the face sheets about their own axis, $\left(\frac{E_t \cdot bt^3}{6}\right)$, reduces $D$ to $\left(\frac{E_t \cdot btd^2}{2}\right)$, and thus the expression for the shear stress distribution in the core reduces to the simplest form:

$$\tau = \frac{V}{bd}$$  \hspace{1cm} (5.10)

The shear stress distribution still remains constant in the core and now varies linearly in the faces. Figure 5.2 illustrates the regression of a parabolic core shear stress distribution to a constant value in the core. For a simply supported beam subjected to a concentrated
point load \( P \) at midspan, the shear force \( V \) is represented by \( \frac{P}{2} \), thus the shear stress in the core can be calculated as:

\[
\tau = \frac{P}{2bd}
\]  

\( (5.11) \)

### 5.2.2 DEFLECTION OF A SIMPLY SUPPORTED SANDWICH BEAM

The transverse displacement of a composite sandwich beam can be separated into two independent components: deformation due to bending (\( \Delta_B \)) and deformation due to shear (\( \Delta_S \)), as shown in Figure 5.3. The deflection associated with bending (\( \Delta_B \)), can be calculated from the ordinary beam theory relationship:

\[
\frac{d^2 \Delta_B}{dx^2} = \frac{M(x)}{D}
\]

\( (5.12) \)

where \( M(x) \) is the bending moment at a distance (\( x \)) from the support along the beam, and \( D \) is the flexural rigidity of the composite sandwich element from the previous discussion. For a simply supported beam with a concentrated point load \( P \) at midspan, double integration of the linear moment relationship, leads to the midspan deflection equation:

\[
\Delta_B = \frac{PL^3}{48D}
\]

\( (5.13) \)

Here \( D \) includes the flexural rigidity of the face sheets about their own elastic neutral axis and is given as:

\[
D = E_t \cdot \frac{bt^3}{6} + E_t \cdot \frac{btd^2}{2}
\]

\( (5.14) \)
The deflection associated with shear ($\Delta_s$) may be calculated from the relationship between the slope of the beam:

$$\theta_s = \frac{d\Delta_s}{dx} \quad (5.15)$$

and the core shear strain:

$$\gamma = \frac{\tau}{G} = \frac{V}{Gb\gamma} \quad (5.16)$$

as illustrated in the sandwich diagram in Figure 5.4. The distance from d to e is equal to $d \cdot \theta_s$; it is also equal to the distance from c to f, which is in turn equal to $\gamma c$. Thus, the following equalities can be established:

$$\frac{d\Delta_s}{dx} = \frac{\gamma c}{Gb \gamma} = \frac{V(x)}{Gb d} = \frac{V(x)}{AG} \quad (5.17)$$

where

$$A = \frac{bd^2}{c} \quad (5.18)$$

The quantity A is termed the shear area whereas the product AG is often referred to as the shear stiffness of the sandwich, which is somewhat analogous to EI, the flexural stiffness of a sandwich element. For a simply supported beam with a concentrated point load $P$ applied at the midspan, single integration of the equation:

$$\frac{d\Delta_s}{dx} = \frac{V(x)}{AG} \quad (5.20)$$

results in the maximum displacement equation which also occurs at midspan.

$$\Delta_s = \frac{PL}{4AG} \quad (5.21)$$
5.3 ADVANCED SANDWICH THEORY

Advanced Sandwich Theory differs from Elementary Sandwich Theory in the realization that the local bending stiffness of the faces has an effect upon the shear deformation of the core. Referring back to Figure 5.3, Allen states, “it is clear that if the faces and the core are to remain in contact, the faces are being called upon to bend to an infinite curvature at the center of the beam. This is not possible; instead the faces bend locally some distance either side of the center-line of the beam, smoothing out the sharp discontinuity in the shear deflection curve. In doing so, the faces reduce the shear deflection at the expense of introducing additional bending moments and shear forces into the faces (1969).” Under this scenario of assumptions, the equations for bending stresses, shear stresses, and shear deformation are modified by several parameters.

5.3.1 BENDING & SHEAR STRESS MODIFICATIONS

Bending stresses at the extreme fibers of the face sheets can be calculated by:

$$\sigma = \frac{PL}{4} \left( \frac{c + 2t}{2I} \psi_2 + \frac{t}{2I_f} (1 - \psi_3) \right)$$ (5.22)

where $I_f$ represents the local bending stiffness of the face sheets. Maximum shear stress in the core is calculated by:

$$\tau = \frac{P}{2bd} \left( 1 - \frac{I_f}{I} \right) \psi_2$$ (5.23)

The parameters $\psi_2$ and $\psi_3$ are calculated by:

$$\psi_2 = 1 - \sqrt{1 - (\beta_1)^2}$$ (5.24)

$$\psi_3 = 1 - \frac{\beta_1}{\theta}$$ (5.25)
where:

$$\beta_i = \frac{\sinh \theta - (1 - \cosh \theta) \tanh \varphi}{\sinh \theta \tanh \varphi + \cosh \theta}$$  \tag{5.26}$$

where:

$$\theta = \frac{L}{c} \left[ \frac{G_c}{2E_f} \frac{c}{t} \left( 1 + \frac{3d^2}{t^2} \right) \right]^{\frac{1}{3}}$$  \tag{5.27}$$

and

$$\varphi = L_1 \left( \frac{AG_c}{E_f I_f \left( 1 - \frac{I_f}{I} \right)} \right)^{\frac{1}{3}}$$  \tag{5.28}$$

$L_1$ is the length of the overhang of the sandwich beam at each end, and for the panels investigated in this research is always equal to 2.0 in.

### 5.3.2 SHEAR DEFLECTION COMPONENT MODIFICATION

The component of deformation associated with bending for a simply supported beam subjected to a concentrated point load at midspan remains unchanged. However, the associated shear deformation at midspan is reduced as shown by the equation:

$$\Delta_s = \frac{PL}{4AG_c} \left( 1 - \frac{I_f}{I} \right)^2 \psi_1$$  \tag{5.29}$$

where:

$$\psi_1 = 1 - \frac{\sinh \theta + \beta_i (1 - \cosh \theta)}{\theta}$$  \tag{5.30}$$

For a detailed derivation of these formulae, refer to Allen’s book “The Analysis and Design of Structural Sandwich Panels” pages 21-32.
5.4 MODELING IMPLEMENTATION IN MATLAB

Both Elementary Sandwich Theory (EST) and Advanced Sandwich Theory (AST) were incorporated into a MATLAB code aimed to predict the flexural response of various sandwich panel configurations and span lengths. EST and AST were both derived for linear elastic materials, which is clearly not applicable to the current sandwich panels under investigation. Therefore, the MATLAB program was designed to take a piecewise linear approach in an effort to approximate the material nonlinearity. It does, however, neglect geometric nonlinearity which occurs under large deflection scenarios. Thus, a Finite Element approach would be more appropriate to incorporate the nonlinear geometric effects. ANSYS, the selected commercially available Finite Element Modeling software, does not have an integrated orthotropic material model which possesses a nonlinear shear stress-strain relationship. It is possible to develop such a custom material model using the USERMAT programmable feature; however this requires relatively extensive programming knowledge and time. Since the scope of this project was more concerned with material properties, the MATLAB approach was used. A copy of the MATLAB modeling code can be found in Appendix B.

5.5 ANALYTICAL RESULTS

Due to the extensive amount of flexural test data, the analysis was performed for 3 in. wide beams with “checkerboard” through-thickness fiber configuration pattern 5, of varying span lengths of 28, 40, 52, and 64 in. tested in the direction perpendicular to the continuous walls. “Checkerboard” insertion pattern 5 was chosen for it has the highest shear stiffness out of all tested patterns and similar shear characteristics in perpendicular directions. The measured and predicted load versus midspan deflection results for the
four spans 28, 40, 52, and 64 in. using the Elementary and Advanced Sandwich Theories are shown in Figures 5.5 through 5.12 respectively. The analysis was based on the piecewise approximation of the measured tensile and shear stress versus strain results. The analysis illustrates the dominate effect of the shear material nonlinearity upon the flexural behavior. The piecewise linear approach is able to capture the nonlinear shear effect and provide a reasonably accurate flexural prediction at lower levels of lateral displacement. The Advanced Sandwich Theory formulation appears to be more appropriate for shorter span lengths of 28 and 40 in., while Elementary Sandwich Theory is sufficient for longer span lengths of 52 and 64 in. The deviation observed in the initial linear portion of the load versus midspan deflection relationship could be due to the lack of sensitivity of the potentiometers to measure readings at low levels of deformation as previously noted in Chapter 4.

Thus, an improved power function of the form:

\[
\gamma = \gamma_o \left( \frac{1}{n} \left( \frac{\tau}{\tau_o} \right)^n - \frac{1}{n} + 1 \right)
\]

where \( \gamma_o, \tau_o, \) and \( n = 1 \) define the initial linear portion of the shear stress-strain relationship and the curve factor \( n > 1 \), which defines the latter degree of nonlinearity, was fit using a solver routine in Excel to the average shear stress versus shear strain results. The curve fitting performed is illustrated in Figure 5.13, where \( \gamma_o = 0.01276 \) (in/in), \( \tau_o = 36.576 \) (psi), and \( n = 7.496 \). The measured and predicted load versus midspan deflection results using this constitutive modeling approach for the shear data for the four spans 28, 40, 52, and 64 in. using the Elementary and Advanced Sandwich Theories are shown in Figures 5.14 through 5.21. This refined approach predicted the
load versus midspan deflection behavior more accurately than the previous method at low levels of deformation. Although, due to some inaccuracy in the shear curve fitting, the nonlinear portion of the load versus midspan deflection prediction was affected. However, the initial linear portion is more representative of real world structural design guidelines and deflection criteria under service load conditions.

Another important consideration in the analytical phase of this research is the relative amounts of deformation due to bending and shear for various span lengths. Figures 5.22 through 5.29, illustrate the percentages of deflections due to bending and shear predicted by Elementary and Advanced Sandwich Theories for the four span lengths considered in this case. Since the initial portion of the load versus midspan deflection relationship is linear, the relative contributions of the individual deflection components remain constant. However, as the level of deformation increases, shear becomes the dominate contribution to midspan deflection as predicted by the sandwich theories. Figure 5.30 and Figure 5.31, predicted by Elementary and Advanced Sandwich Theories, illustrate the effect of the span length-to-depth ratio to the relative amounts of deformation for the initial linear portion of the load versus midspan deflection relationship. The figures indicate that the contribution of shear deformation to the total deflection linearly decreases as the span length/depth ratio increases, and becomes almost 50 percent of the total deformation for the span length/depth ratio of 35 for the sandwich panels considered in this investigation. Furthermore, as the span length/depth ratio approaches 60, the shear deformation term becomes negligible. These results suggest the necessity to increase the core shear modulus considerably in order to use the proposed sandwich panels in short span, high demand structural applications.
5.6 SUMMARY AND CONCLUSIONS

Experimental results have shown the in-plane face sheet tensile properties are not significantly affected by through-thickness fiber insertion pattern nor the test direction. Ultimate strength limits and fiber rupture failure modes are slightly affected. Fiber rupture tends to occur through the zones of imperfection created by the through-thickness fiber insertion patterns. The axial stress versus strain response is bilinear in nature due to the presence of the fibers in the perpendicular directions.

The shear behavior of the proposed GFRP sandwich panels is highly dependent on 3-D fiber insertion pattern and testing direction. Non-wall shear is exemplified by testing of regular array patterns in both directions in addition to “continuous” wall patterns tested only in the direction perpendicular to the wall insertions. Wall shear is exemplified by testing of “continuous” wall patterns in the direction parallel to the continuous walls and “checkerboard” patterns tested in both directions. Wall type shear is always stiffer than non-wall shear for a given fipsi, however, a higher degradation in stiffness is exhibited by wall shear. The overall shear stress versus shear strain response is initially linear up to the initiation of the first crack in the polyurethane foam, followed by a nonlinear behavior with degradation in stiffness as cracking propagates throughout the foam and plastic hinges form at each end of the through-thickness fibers. For wall shear, at significant levels of shearing strain, an increase in stiffness occurs due to the 3-D fibers now being subjected to axial tensile forces.

The flexural performance of the sandwich panels is primarily dominated by the core shear behavior. The nonlinear load versus midspan deflection relationship is similar in mathematical nature to that of shear and can be predicted quite accurately with a piecewise linear approximation approach using Elementary and Advanced Sandwich
Theories. Accurate modeling of shorter span members requires using the Advanced Sandwich Theory formulation with a piecewise linear approximation of the actual shear stress-strain relationship. Longer span members are accurately modeled using the Elementary Sandwich Theory formulation with a piecewise linear approximation of an improved power function fit to the actual shear stress-strain relationship. Analytical results have shown that the relative amount of shear deflection in the initial linear portion of the load versus midspan deflection relationship linearly decreases as span length/depth ratio increases. Only at large span length/depth ratios, roughly greater than 35, the shear contribution is less than bending deformation. These results suggest an important need to increase the core shear modulus significantly in order to use the proposed panels in current structural applications.

5.7 SUGGESTIONS FOR FUTURE WORK

Significant improvement of the shear stiffness of the composite panels could be obtained by inserting the 3-D fibers at an angle of roughly 45°, where truss type action could be achieved. This idea is strengthened by the sudden increase in shear stiffness observed at high levels of shear strain. The ability of the proposed sandwich panels to undergo rather significant amounts of deformation suggests high energy absorption capabilities and thus a potential market for them in infrastructure blast protection, automobile crash improvement, and perhaps new guardrail designs. More advanced research studies, specifically high strain-rate, impact, and fatigue tests would need to be conducted in order to quantify the feasibility of these applications. In addition, these panels are a viable candidate for more advanced, nonlinear Finite Element Analysis. A
current endeavor involves developing a macro to predict the shear stress versus shear strain response based on 3-D fiber configuration and density. However, a user defined orthotropic material model with nonlinear shear capabilities is in need of development in order to use commercial Finite Element software in flexural predictions. Finally, experimental research needs to be extended to two dimensions where testing is performed on actual composite panels. These test results in conjunction with knowledge gained from previous Finite Element studies would result in the extension to advanced plate modeling techniques.
Figure 5.1 Sandwich Beam Element Dimension Illustration

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Figure 5.8 Advanced Sandwich Prediction vs. Actual Flexural Behavior for 3 in. Wide 40” Span Length Sandwich Beam of Core Pattern 5 Tested in x-direction
Figure 5.9 Elementary Sandwich Prediction vs. Actual Flexural Behavior for 3 in. Wide 52” Span Length Sandwich Beam of Core Pattern 5 Tested in x-direction

Figure 5.10 Advanced Sandwich Prediction vs. Actual Flexural Behavior for 3 in. Wide 52” Span Length Sandwich Beam of Core Pattern 5 Tested in x-direction
Figure 5.11 Elementary Sandwich Prediction vs. Actual Flexural Behavior for 3 in. Wide 64” Span Length Sandwich Beam of Core Pattern 5 Tested in x-direction

Figure 5.12 Advanced Sandwich Prediction vs. Actual Flexural Behavior for 3 in. Wide 64” Span Length Sandwich Beam of Core Pattern 5 Tested in x-direction
Figure 5.13 Average Shear Stress versus Shear Strain Curve Fitting

\[ \gamma = \gamma_o \left( \frac{1}{n} \left( \frac{\tau}{\tau_o} \right)^n - \frac{1}{n} + 1 \right) \]

\[ \gamma_o = 0.01276 \]

\[ \tau_o = 36.576 \]

\[ n = 7.496 \]
Figure 5.14 Elementary Sandwich Prediction vs. Actual Flexural Behavior for 3 in. Wide 28” Span Length Sandwich Beam of Core Pattern 5 Tested in x-direction

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Figure 5.23 Advanced Sandwich Theory - % Shear & Bending Deflection for 3 in. Wide 28” Span Length Sandwich Beam of Core Pattern 5 Tested in x-direction
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Figure 5.25 Advanced Sandwich Theory - % Shear & Bending Deflection for 3 in. Wide 40” Span Length Sandwich Beam of Core Pattern 5 Tested in x-direction
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Figure 5.29 Advanced Sandwich Theory - % Shear & Bending Deflection for 3 in. Wide 64” Span Length Sandwich Beam of Core Pattern 5 Tested in x-direction
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Figure 5.31 **AST** Span Length/Depth Ratio versus % Shear & Bending Deflections
REFERENCES


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Figure A.2  Tension Stress vs. Strain Results for T-(RDPS4-4)-y-#
Figure A.3  Tension Stress vs. Strain Results for T-(RDPS4-5)-x-#

Figure A.4  Tension Stress vs. Strain Results for T-(RDPS4-5)-y-#
Figure A.5  Tension Stress vs. Strain Results for T-(RDPS4-6)-x-

Figure A.6  Tension Stress vs. Strain Results for T-(RDPS4-6)-y-
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Figure A.8  Shear Stress vs. Shear Strain Results for S-(RDPS4-2)-2.0-2-4-x-#
Figure A.9  Shear Stress vs. Shear Strain Results for S-(RDPS4-2)-2.0-2-4-y#

Figure A.10  Shear Stress vs. Shear Strain Results for S-(RDPS4-3)-2.0-3b-4-x#
Figure A.11  Shear Stress vs. Shear Strain Results for S-(RDPS4-3)-2.0-3b-4-y-#

Figure A.12  Shear Stress vs. Shear Strain Results for S-(RDPS4-3)-2.0-3b-4-y-##
Figure A.13  Shear Stress vs. Shear Strain Results for S-(RDPS4-4)-2.0-4-4-x#

Figure A.14  Shear Stress vs. Shear Strain Results for S-(RDPS4-4)-2.0-4-4-y#
Figure A.15  Shear Stress vs. Shear Strain Results for S-(RDPS4-4)-2.0-4-4-y-

Figure A.16  Shear Stress vs. Shear Strain Results for S-(RDPS4-5)-2.0-5-4-x-#
Figure A.17  Shear Stress vs. Shear Strain Results for S-(RDPS4-5)-2.0-5-4-y#

Figure A.18  Shear Stress vs. Shear Strain Results for S-(RDPS4-6)-2.0-6-4-x#
Figure A.19  Shear Stress vs. Shear Strain Results for S-(RDPS4-6)-2.0-6-4-y#

Figure A.20  Load vs. Midspan Deflection Results for F-(RDPS4-4)-2.0-4-4-x-#-3
Figure A.21  Load vs. Midspan Deflection Results for F-(RDPS4-4)-2.0-4-4-y-#-3

Figure A.22  Load vs. Midspan Deflection Results for F-(RDPS4-4)-2.0-4-4-x-#-6
Figure A.23  Load vs. Midspan Deflection Results for F-(RDPS4-4)-2.0-4-4-y-#-6

Figure A.24  Load vs. Midspan Deflection Results for F-(RDPS4-5)-2.0-5-4-x-#-3
Figure A.25  Load vs. Midspan Deflection Results for F-(RDPS4-5)-2.0-5-4-y-#-3

Figure A.26  Load vs. Midspan Deflection Results for F-(RDPS4-5)-2.0-5-4-x-#-6
Figure A.27  Load vs. Midspan Deflection Results for F-(RDPS4-5)-2.0-5-4-y-#-6

Figure A.28  Load vs. Midspan Deflection Results for F-(RDPS4-6)-2.0-6-4-x-#-3
Figure A.29  Load vs. Midspan Deflection Results for F-(RDPS4-6)-2.0-6-4-y-#-3

Figure A.30  Load vs. Midspan Deflection Results for F-(RDPS4-6)-2.0-6-4-x-#-6
Figure A.31  Load vs. Midspan Deflection Results for F-(RDPS4-6)-2.0-6-4-y-#-6
APPENDIX B
clc
clear
%-----------------------------------Program Begins-------------------------
load Geometry.txt;
load Shear.txt;
load Tension.txt;
load Flexure.txt;
L=Geometry(1);
b=Geometry(2);
h=Geometry(3);
t=Geometry(4);
d=Geometry(5);
c=Geometry(6);
% plot(Shear(:,2),Shear(:,1));
% figure;
% plot(Tension(:,1),Tension(:,2));
% figure;
% plot(Flexure(:,2),Flexure(:,1));
% figure;

%--------------------------------Calculation Sequence----------------------
%-----------------------------------Shear Segment--------------------------

Tau=Shear(:,1);
Gamma=Shear(:,2);
count=1;
for i=1:length(Gamma)
    if(Gamma(count)<0.01)
        count=count+1;
    end
end

temp=1;
for i=1:count
    if(Tau(temp)<5)
        temp=temp+1;
    end
end
for i=1:count-temp
    G(i)=Gamma(i+temp);
    T(i)=Tau(i+temp);
end
[G1,s]=polyfit(G,T,1);
count1=count;

for i=count+1:length(Gamma)
if(Gamma(count)<0.03)
    count=count+1;
end
end
l=1;
for i=count1+1:count
    GG(l)=Gamma(i);
    TT(l)=Tau(i);
    l=l+1;
end
[G2,s]=polyfit(GG,TT,1);
disp('G1 Value')
G1(1)
disp('G2 Value')
G2(1)

%---------------------------------Tension Segment-----------------------------
Epsilon=Tension(:,1);
Sigma=Tension(:,2);

count=1;
for i=1:length(Epsilon)
    if(Epsilon(count)<0.0005)
        count=count+1;
    end
end
temp=1;
for i=1:length(Epsilon)
    if(Epsilon(temp)<0.003)
        temp=temp+1;
    end
end
j=1;
for i=count:temp
    E(j)=Epsilon(i);
    S(j)=Sigma(i);
    j=j+1;
end
[E1,s]=polyfit(E,S,1);

count1=temp;
count=1;
for i=1:length(Epsilon)
if(Epsilon(count)<0.008)
    count=count+1;
end
end

l=1;
for i=count1+1:count
    EE(l)=Epsilon(i);
    SS(l)=Sigma(i);
    l=l+1;
end
[E2,s]=polyfit(EE,SS,1);
disp('E1 Value')
E1(1)
disp('E2 Value')
E2(1)

%*********************Elementary Sandwich Theory*********************

I= (b*t^3/6)+(b*t*d^2/2);
If=(b*t^3/6);
A= (b*d^2/c);
L1=2;

P_act=Flexure(:,1);
del_act=Flexure(:,2);
max_p= max(P_act);
max_d=max(del_act);
n_inc=100;
p_inc=max_p/n_inc;

%---------------------NonLinear Piecewise Approximation---------------------
vals=1;
for i=1:n_inc
    P_i(i)=i*p_inc;
    Tau_c(i)=P_i(i)/(2*b*d);
    Sigma_f(i)=(P_i(i)*L*h)/(8*I);
    % Gamma=Shear(:,2);
    count=1;
    for j=1:length(Gamma)
        if (Tau(j)<Tau_c(i))
            count=count+1;
        end
    end
end
if (count==1)
    if (Tau(count)==0)
        G_val(i)=0;
    else
        G_val(i)=abs(Tau(count)/Gamma(count));
    end
else
    G_val(i)=abs(Tau(count)+Tau(count-1)/(Gamma(count)+Gamma(count-1)));
end
% Epsilon=Tension(:,1);
count=1;
for j=1:length(Epsilon)
    if (Sigma(j)<Sigma_f(i))
        count=count+1;
    end
end
if (count==1)
    if (Sigma(count)==0)
        E_val(i)=0;
    else
        E_val(i)=abs(Sigma(count)/Epsilon(count));
    end
else
    E_val(i)=abs((Sigma(count)+Sigma(count-1))/(Epsilon(count)+Epsilon(count-1)));
end

val=P_i(i);
if (E_val(i)==0)
    del_b(i)=0;
else
    del_b(i)=val*(L^3)/(48*E_val(i)*I);
end

if (G_val(i)==0)
    del_s(i)=0;
else
    del_s(i)=val*L/(4*A*G_val(i));
end

del_tot(i)=del_b(i)+del_s(i);

if (del_tot(i)>=max_d)
    break
end
if(del_tot(i)<=0.48)
    vals=vals+1;
end
end

%----------------------------------------------------------------------------------------------------------

n=50;
store1=numel(del_tot);

for i=1:n
    vala=1;
    for j=1:store1
        if(del_tot(j)<=i*max_d/n)
            vala=vala+1;
        end
    end
    Pdel_ba(i)=del_b(vala)/del_tot(vala);
    Pdel_sa(i)=del_s(vala)/del_tot(vala);
    xa(i)=del_tot(vala);
end

plot(xa,Pdel_ba,'b-',xa,Pdel_sa,'r-');
xlabel('Deflection (in)')
ylabel('% Deflection')
title('Elementary Sandwich Theory')

disp(' Elementary Percentage Values')

Pdel_b=del_b(vals)/del_tot(vals)
Pdel_s=del_s(vals)/del_tot(vals)
figure;
plot(Flexure(:,2),Flexure(:,1),'r-',del_tot,P_i,'b-');
xlabel('Deflection (in)')
ylabel('Load (lbs)')
title('Elementary Sandwich Theory - Actual vs Prediction')

%******************************************************************************Advanced Sandwich Theory******************************************************************************
vals=1;
for i=1:n_inc
    P_i(i)=i*p_inc;
    Tau_c(i)=P_i(i)/(2*b*d);
    Sigma_f(i)=(P_i(i)*L*h)/(8*I);
    % Gamma=Shear(:,2);
count=1;
for j=1:length(Gamma)
    if (Tau(j)<Tau_c(i))
        count=count+1;
    end
end
if (count==1)
    if(Tau(count)==0)
        G_val(i)=0;
    else
        G_val(i)=abs(Tau(count)/Gamma(count));
    end
else
    G_val(i)=abs(Tau(count)+Tau(count-1)/(Gamma(count)+Gamma(count-1)));
end
% Epsilon=Tension(:,1);
count=1;
for j=1:length(Epsilon)
    if (Sigma(j)<Sigma_f(i))
        count=count+1;
    end
end
if (count==1)
    if(Sigma(count)==0)
        E_val(i)=0;
    else
        E_val(i)=abs(Sigma(count)/Epsilon(count));
    end
else
    E_val(i)=abs((Sigma(count)+Sigma(count-1))/(Epsilon(count)+Epsilon(count-1)));
end
else
    E_val(i)=abs((Sigma(count)+Sigma(count-1))/(Epsilon(count)+Epsilon(count-1)));
end

a=((A*G_val(i))/(E_val(i)*If*(1-(If/I))))^.5;
phi=a*L1;
theta=a*L/2;
beta1=(sinh(theta)-(1-cosh(theta))*tanh(phi))/(sinh(theta)*tanh(phi)+cosh(theta));
psi1=1-((sinh(theta)+beta1*(1-cosh(theta)))/theta);
psi2=1-(1-beta1^2)^0.5;
psi3=1-(beta1/theta);
val=P_i(i);
del_b(i)=val*(L^3)./(48*E_val(i)*I);
del_s(i)=val*L*psi1*(1-If/I)^2/(4*A*G_val(i));
del_tot(i)=del_b(i)+del_s(i);
if (del_tot(i)>=max_d)
break
end
if(del_tot(i)<=0.48)
vals=vals+1;
end
end

%--------------------------------------------------------------------------
n=50;
store1=numel(del_tot);
for i=1:n
    vala=1;
    for j=1:store1
        if(del_tot(j)<=i*max_d/n)
            vala=vala+1;
        end
    end
    end
    Pdel_ba(i)=del_b(vala)/del_tot(vala);
Pdel_sa(i)=del_s(vala)/del_tot(vala);
    xa(i)=del_tot(vala);
end

figure;
plot(xa,Pdel_ba,'b-',xa,Pdel_sa,'r-');
xlabel('Deflection (in)')
ylabel('% Deflection')
title('Advanced Sandwich Theory')
disp(' Advanced Percentage Values')
Pdel_b=del_b(vals)/del_tot(vals)
Pdel_s=del_s(vals)/del_tot(vals)
figure;
plot(Flexure(:,2),Flexure(:,1),'r-',del_tot,P_i,'b-');
xlabel('Deflection (in)')
ylabel('Load (lbs)')
title('Advanced Sandwich Theory - Actual vs Prediction')