DEFLECTION AND CRACK CONTROL OF CONCRETE BEAMS PRESTRESSED BY CFRP BARS

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Abstract: Carbon fibre reinforced plastic, CFRP, reinforcement has been used for prestressing concrete bridges in North America, Europe and Japan for the last ten years. The non-corrosive characteristics, high strength to weight ratio and good fatigue properties of CFRP reinforcement could significantly increase the service life of structures. However, the high cost and low ductility of CFRP reinforcement due to its limited strain at failure could affect the use of this new material in practice. Partially prestressing of concrete members provides solution for these problems due to its advantages in reducing the cost by increasing the eccentricity of the prestressing reinforcement, and improving the deformability. Therefore, deflection and cracking of concrete beams partially prestressed by CFRP reinforcement are of prime importance and need to be investigated.

An experimental program was undertaken at the University of Manitoba to study the serviceability of concrete beams prestressed by CFRP reinforcements. The experimental program consisted of eight concrete beams prestressed by Leadline CFRP bars, produced by Mitsubishi Kasei, Japan and two concrete beams prestressed by conventional steel strands. The beams were 6.2 meter long and 330 mm in depth. The experimental program was conducted to examine the various limit states and flexural behaviour of concrete beams prestressed by CFRP bars.

The measured deflection and crack width are compared to the predicted values based on the American, Canadian and European codes, using the properties of the CFRP bars. The behaviour of beams prestressed by CFRP bars is compared to similar beams prestressed by conventional steel strands. This paper presents theoretical models, proposed to predict deflection and crack width of beams prestressed by CFRP bars under service loading conditions. The crack width of the beams is predicted using appropriate bond factors for this type of reinforcement. Design guidelines for prestressed concrete beams with CFRP reinforcement are presented.

Keywords: Beam, bond, carbon, cracking, deflection, tension stiffening, flexure, FRP, prestressed concrete, serviceability.
Introduction

Carbon fibre reinforced plastic, CFRP, has been successfully used in concrete structures as prestressing reinforcement. Several concrete bridges prestressed by CFRP reinforcement were built in Japan, Europe and Canada. The non-corrosive characteristics of CFRP reinforcement, in addition to its high strength-to-weight ratio, good fatigue properties and low relaxation losses are properties that significantly increase the service life of the structures. The current practice is to keep the jacking stresses within 60% of the nominal tensile strength of FRP prestressing reinforcement [1]. Most of the existing concrete structures prestressed by CFRP are designed to be free from cracks under service loading conditions which means fully prestressed. The concerns regarding the use of CFRP prestressing reinforcements in concrete structures are the low ductility due to its limited strain at ultimate and secondly due to the high cost of the material. Therefore, it is advantageous, in some cases, to allow prestressed members to crack under the full service load, which means partially prestressed members, which in fact have less camber, more ductility and less cost than the fully prestressed members.

CFRP prestressing reinforcement is considered to be the best among the different types of fibre reinforced plastic, FRP, reinforcement commercially available for prestressed concrete structures due its excellent fatigue performance [2]. In general, partial prestressing may be achieved either by adding non-prestressed reinforcement in the section or by lowering the jacking stresses. The technique of using low jacking stresses may be more convenient in this case since it reduces the cost. The key of the design of partially prestressed members is the serviceability of the member, that is, the deflection and the crack width of the member at service loading conditions. Since the elastic modulus of CFRP prestressing reinforcement is lower than the steel, the deformations of members prestressed by CFRP reinforcement, after initiation of cracks, are expected to be greater than similar members prestressed by steel strands. The cracking behaviour is mainly related to the bond of the reinforcement, which significantly influenced by the type of CFRP reinforcement in terms of being strands or bars and the surface texture of being indented or smooth.

This study investigates the flexural behaviour of concrete beams prestressed by Leadline CFRP reinforcement, produced by Mitsubishi Kasei, Japan. The various limit state behaviour considered include deflection before and after cracking, crack pattern, spacing and width, strain distribution and failure modes. An analytical model, based on the compatibility and equilibrium approach, was used to predict the behaviour of beams. This paper also examines the applicability of different methods available to calculate the deflection of beams prestressed by CFRP bars. Effect of bond characteristic of CFRP bars on the tension stiffening of concrete is included. Experimental and computed deflection was compared. The agreement was found to be quite good. A procedure was formulated for predicting the location of the centroidal axis of the cracked sections prestressed by CFRP bars. The crack width of the beams was also predicted using different approaches under service loading conditions. The findings of this experimental study, combined with available information, were used to propose design recommendations of concrete beams prestressed by CFRP bars.
The Experimental Program

The experimental program consists of testing eight concrete beams prestressed by Leadline CFRP bars, in addition to two beams prestressed by conventional steel strands. The beams were 5.8 meter simply supported span with 200 mm projection from each end and 330 mm in depth, which is a typical span to depth ratio used for bridge girders. The cross section of the tested beams was T-section with two flange widths, 200 and 600 mm as shown in Fig.1. The two jacking stresses used for the CFRP bars were 50 and 70 % of the ultimate guaranteed strength of the CFRP bars, reported by the manufacturing company. The level of prestressing and consequently the concrete stress distribution along the section was varied. The distribution of the CFRP bars in the tension zone was also varied to study its effect on the cracking behaviour of the concrete beams prestressed by CFRP bars. A summary of the manufacturing and the testing process of the beams can be found in a separate paper [3].

Flexural Behaviour

The two modes of failure, observed in this study, were rupture of the furthest CFRP bar from the neutral axis and crushing of the concrete at the extreme compression fibre within the constant moment zone. At the onset of rupture of the CFRP bar, a horizontal crack occurred, in some of the beams, at the level of the bar as well as extensive cracks extended to the top flange of the beam. The horizontal cracks occurred due to the release of the elastic strain energy after rupture of bars. The released elastic energy, which is partly absorbed by the concrete, resulted in cracking at the level of the bars [4]. The rupture of the first CFRP bar was followed by a progressive failure of the other CFRP bars. Cracking of the beams, failed by crushing of concrete, was not as extensive as the cracks occurred for beams failed by rupture of the CFRP bars. No slip of the CFRP bars was observed to any of the two ends of the tested beams. A comparison of the behaviour of beams prestressed by CFRP bars and steel strands is presented in references [3,5]. The effect of partial prestressing on the behaviour of the beams is studied in the following section.
Beams prestressed by CFRP jacked to 70% of the guaranteed strength, which are considered to be fully prestressed beams, exhibited 4 to 8% higher ultimate strength than beams partially prestressed by CFRP jacked to 50% of the guaranteed. However, the deflection at failure of beams with a jacking stress equivalent to 50% of the guaranteed strength was about 13 to 22% higher, as shown in Fig.2. The mode of failure of the beams with different jacking stresses was the same. Concrete strains at failure were higher for the beams jacked to 50% of the guaranteed strength, as shown in Fig.3. This is attributed to the lower initial CFRP strain at jacking. For beams with large flange width, the neutral axis depth at ultimate was about 25% less for partially prestressed beam than the fully prestressed one. While the neutral axis depth of the beams with small flange width was almost the same for both partially and fully prestressed beams.

![Figure 2: Load-Deflection of Partially and Fully Prestressed Beams with CFRP Bars](image)

![Figure 3: Concrete strain at Ultimate](image)
The crack pattern of beams with 600 and 200 mm flange width partially prestressed by different number of CFRP bars is shown in Fig. 4. The crack distribution within the constant moment zone is given for maximum crack width of 0.4 mm, which is considered in this investigation corresponds to the full service load level. Beams prestressed by low reinforcement ratio, had fewer cracks and consequently larger average crack spacing. This may be attributed to the higher increase in the CFRP stresses for these beams than beams with higher reinforcement ratio and consequently more bond damage occurred in the vicinity of each crack right after the initiation of the first crack.

Test results indicated that stabilization of the flexural cracks occurred at significantly higher strain level for beams with low reinforcement ratio. Stabilization of crack pattern is defined by the stage when the number of cracks did not increase by increasing the applied load [6]. Again, this may be attributed to the sudden and high increase in the stresses after cracking for beams with low reinforcement ratio and consequently the larger debonded zone near each crack.

Fig. 4 Crack Distribution for Beams with Different Reinforcement Ratios
Analysis of the Test Results

Cracking and ultimate resistance of the tested beams were predicted as given in Table 1. The designation of the beams have the first letter either T, R, or S, refers to T-section with flange width of 600 mm, Rectangular section with flange width of 200 mm and Steel reinforcement respectively. The first number of the beam designation is either 2 or 4 refers to the number of prestressing rods, while the second number, .5 or .7, refers to the ratio of the jacking to the ultimate guaranteed strength. The last letter in the beam designation, H or V, refers to the configuration of the bars in the tension zone, either Horizontal or Vertical. The cracking moment, \( M_{cr} \), was predicted using two different methods using a rupture strength of the concrete, \( f'_{r} \), according to the CSA Code [7]. In the first method, the equilibrium and compatibility conditions were applied. For each incremental strain, the neutral axis depth was obtained by trial and error. While the second method was based on the classical analysis of prestressed sections using the effective prestressing force, \( P_{e} \), gross cross sectional area of concrete, \( A_{g} \), eccentricity of the prestressing reinforcement, \( e \), and section modulus, \( S_{b} \), as given by equation (1). The predicted cracking load was overestimated by 22 and 9 % using the first and the second methods respectively. The ratio of the predicted to the observed cracking load of the beams with low reinforcement ratio was high in comparison to beams with high reinforcement ratio. This may be attributed to the increase in shrinkage cracks in beams prestressed by less reinforcement.

\[
M_{cr} = P_{e} \varepsilon + \left[ f'_{r} + \frac{P_{e}}{A_{g}} \right] S_{b}
\]

Table 1 Comparison Between the Predicted and the Experimental Results

<table>
<thead>
<tr>
<th>Beam</th>
<th>( P_{cr,exp} ) (kN)</th>
<th>( P_{cr,prd} ) (kN)</th>
<th>( P_{cr,prd} ) method #1</th>
<th>( P_{cr,prd} ) method #2</th>
<th>( P_{u,exp} ) (kN)</th>
<th>( P_{u,prd} ) (kN)</th>
<th>( P_{u,prd}/exp )</th>
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</thead>
<tbody>
<tr>
<td>T-4-.5-H</td>
<td>25.31</td>
<td>30.85</td>
<td>27.71</td>
<td>1.22</td>
<td>1.09</td>
<td>106.2</td>
<td>97.9</td>
</tr>
<tr>
<td>R-4-.5-H</td>
<td>23.40</td>
<td>24.03</td>
<td>21.49</td>
<td>1.03</td>
<td>0.92</td>
<td>89.3</td>
<td>89.7</td>
</tr>
<tr>
<td>T-4-.5-V</td>
<td>27.33</td>
<td>30.50</td>
<td>27.16</td>
<td>1.11</td>
<td>0.99</td>
<td>97.9</td>
<td>86.1</td>
</tr>
<tr>
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<td>1.40</td>
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<td>1.31</td>
<td>56.8</td>
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<td>77.1</td>
<td>71.4</td>
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<tr>
<td>S-R-2-.5</td>
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<td>29.32</td>
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<td>1.05</td>
<td>0.93</td>
<td>70.1</td>
<td>70.8</td>
</tr>
</tbody>
</table>
The ultimate load was predicted using an iteration process which satisfy the equilibrium of the section for every strain increment at the extreme compression fibre. CFRP bars were assumed to have a linear stress-strain relationship with a tensile strength of 2950 MPa based on the measured tensile strength using concrete anchorage [3] and elastic modulus of 147 GPa. The results show underestimation of the ultimate resistance for beams prestressed by CFRP bars. This indicates that the actual strength of the Leadline bars is higher than the measured strength from tension test. Based on the measured ultimate load of the tested beams, CFRP bars have an average ultimate strength of 3230 MPa with a standard deviation of 100 MPa.

**Deflection Prediction**

Deflection of beams prestressed by CFRP reinforcement was predicted using four different methods taking into consideration tension stiffening of concrete. The results obtained from the analysis were in a good agreement with the experimental results. The deflection was calculated based on integration of the mean curvature along the span of the beam for the first three methods. The mean curvature was calculated in the first method using the effective moment of inertia $I_e$ approach developed originally by Branson [8] and modified by Tadros et al. [9]. In the second method, the approach of the CEB-FIP Code [10] was used, where the mean curvature was calculated by interpolating between the cracked and the uncracked curvatures. The curvature was calculated as the slope of the strain profile in the third method. The proposed stress distribution within the cross section is shown in Fig.5. The tensile stresses in concrete were introduced in two locations on the tension side of the beam, below the neutral axis where the tensile stresses in concrete increases linearly up to the rupture strength and secondly on the effective embedment zone around the reinforcement using the concrete tensile stress-strain relationship reported by Collins and Mitchell [11]. The concrete was represented by a parabolic stress-strain relationship in compression. Bond factors involved in the second and the third methods were calculated using the measured deflection.

![Stress Distribution within the Cross Section Accounting for Tension Stiffening](image-url)
In the fourth method, the deflection was calculated using equation (2) where the effective moment of inertia, $I_e$, is calculated using equation (3).

$$\Delta = - k_p \frac{P_e (d_p - y_e) L^2}{E_c I_e} + k_s \frac{M_s L^2}{E_c I_e}$$  \hspace{1cm} (2)$$

$$I_e = \psi^3 I_g + \left( 1 - \psi^3 \right) I_{cr} \leq I_g$$  \hspace{1cm} (3)

where,

$$\psi = \left( \frac{M_{cr} - M_{dc}}{M_s - M_{dc}} \right)$$  \hspace{1cm} (4)

where,

$k_p$ = factor for the calculation of the deflection due to the prestressing reinforcement, depends on the shape of the strand

$k_s$ = factor for the calculation of the deflection due to the applied loads depends on the shape of the loading and boundary conditions

$M_{cr}$ = cracking moment

$M_{dc}$ = decompression moment

$M_s$ = moment at service load

After cracking and due to tension stiffening, the distance of the centroidal axis of the section from the extreme compression fibre $y_e$ has an intermediate value between the cracked and uncracked centroidal distances $y_{cr}$ and $y_g$ respectively. Based on the measured concrete strains at the top surface of the beams and at the level of the prestressing bars, the neutral axis depth was calculated and consequently the effective centroidal distance, $y_e$. Based on this study, equation (5) is proposed to calculate the effective centroidal distance $y_e$ for beams prestressed by CFRP bars.

$$y_e = \psi^2 y_g + \left( 1 - \psi^2 \right) y_{cr} \leq y_g$$  \hspace{1cm} (5)

The percentage of the predicted deflection up to failure load, within 20% error from the measured values, were 95, 93 and 93%, for the first three described methods, respectively, and 82% using the simplified method, (method 4), as shown in Fig.6. Ignoring the tension stiffening effect, 56% of the predicted deflection was within 20% error as shown in Fig.6. Within the service load range, the accuracy of the predicted deflections were 86, 83, 87 and 57% for the four methods respectively. This accuracy is also based on 20% margin of error between the measured and the predicted deflections. The analysis suggests that the calculation of the deflection based on integration of the curvature along the span using any of the three methods gives an
excellent correlation with the measured deflection. The simplified method also provides a good tool for preliminary design. It should be noted that bond factors are required for the second and third methods, while equivalent moment of inertia, $I_e$, method is applicable to any type of reinforcement. Fig. 7 shows the predicted load-deflection of a beam prestressed by CFRP with and without accounting for tension stiffening.

Fig. 6 Percentage of the Predicted Deflection up to Failure Load within 20% Error

Fig. 7 Predicted Deflection of Beam Prestressed by CFRP Bars
Crack Width

Cracks in concrete structures prestressed by steel are normally controlled to minimize the corrosion of steel. Replacing the steel by FRP reinforcement provides an excellent solution for the classical corrosion problem of concrete structures as they are non-corrosive. However, there are many other good reasons for controlling widths of cracks other than corrosion protection. Control of the crack width could be required to avoid leakage or to avoid to impair the appearance of the structure. It is reported that cracks wider than 0.25 to 0.3 mm could lead to public concern [12].

Many formulae are currently available to calculate crack width [10,13,14] of concrete structures. Due to the local and random occurrence of cracks, the level of accuracy of crack width prediction of reinforced or prestressed concrete structures is quite less than the accuracy of deflection prediction [12]. The various parameters affecting the cracking behaviour are the reinforcement stress or strain after decompression, effective concrete area in tension, area of reinforcement, concrete cover and bond characteristics of the reinforcement. There is no universal agreement on the mathematical relationship relating these variables with the crack width. For example, the effective area of concrete in tension presented by Suri and Dilger equation [13] has a square root relationship and a cubic root relationship in Gergely and Lutz equation [14], while it is raised to the power of one in the CEB-FIP Code 1978. Moreover, the definition of concrete area in tension is proposed to be the area below the neutral axis by Suri and Dilger equation, while considered in the CEB-FIP Code and in Gergely and Lutz equation as the effective area of concrete in tension around the reinforcement.

Influence of Major Parameters

The relationship between the major Parameters and the measured crack width of beams prestressed by CFRP bars, is examined. The relationship between the steel stress, after decompression, $f_s$, to the crack width of beams prestressed by steel wires or strands with or without non-prestressed deformed steel bars proposed by Suri and Dilger [13] is given in equation (6). Using the same approach, the stress of the CFRP bars, after decompression, $f_f$, was found to be related to the maximum measured crack width by equation (7), as shown in Fig.8.

$$w_{max} = 0.109 \times 10^{-3} f_s^{1.38} - 0.01 \quad \text{beams prestressed by steel} \quad (6)$$

$$w_{max} = 0.525 \times 10^{-3} f_f^{1.13} - 0.023 \quad \text{beams prestressed by Leadline} \quad (7)$$

Since the stress in the reinforcement is typically related to the crack width in linear function by most of the researchers, a linear regression was performed for beams prestressed by CFRP and compared to the data given, by Suri and Dilger [13], for beams prestressed by steel strands in equations (8) and (9). The linear relationship of the crack width and the reinforcement stress provided a standard error of 0.07 for beams prestressed
by Leadline in comparison to a value of 0.10 for beams prestressed by steel strands.

\[ w_{\text{max}} = 0.116 \times 10^{-2} f_s \quad \text{beams prestressed by steel} \quad (8) \]

\[ w_{\text{max}} = 0.124 \times 10^{-2} f_f - 0.053 \quad \text{beams prestressed by Leadline} \quad (9) \]

![Graph showing measured crack width vs. stresses in CFRP bars](image)

**Fig. 8 Measured Crack Width vs. Stresses in CFRP Bars**

The concrete area in tension, \( A_t \), which considered another major parameter, is related to the crack width as shown in Fig. 9 for beams prestressed by Leadline. Despite of scattering of the results of all the tested beams, the non-linear regression provided equation (10) which compared well to equation (11) proposed for beams prestressed by steel [13].

\[ w_{\text{max}} = 0.033 \left( \frac{A_f}{A_t} \right)^{0.4} \quad \text{beams prestressed by Leadline} \quad (10) \]

\[ w_{\text{max}} = 0.017 \left( \frac{A_s}{A_t} \right)^{0.5} \quad \text{beams prestressed by steel} \quad (11) \]
where,
\[ A_f = \text{area of FRP in tensile zone} \]
\[ A_s = \text{total area of steel in tensile zone} \]

![Graph showing correlation between crack width and reinforcement ratio](image)

**Fig. 9 Correlation Between the Crack Width and \( A_f/A_t \)**

### Crack Width Prediction

The crack width was calculated for all the tested beams prestressed by CFRP bars using three methods given by Suri and Dilger [13], CEB-FIP Code 1987 [14] and Gergely and Lutz [15]. Since bond characteristics of the reinforcement considered to be one of the most important parameters affecting the cracking behaviour of partially prestressed beams, the three methods were used to determine the equivalent bond factors for CFRP bars. The bond factors were found to be different from steel reinforcements since it significantly affected by the surface conditions (smooth or ribbed) and type of reinforcement (strand or bar). The other two factors which could also affect the bond characteristics are the elastic modulus and poisson's ratio [3]. The following subsections discuss each of the above methods.

**Method #1: Suri and Dilger [13]:** The proposed equation (12) was adapted by the CPCI Design Manual [16] to estimate the maximum crack width at the beam soffit. The bond factor \( k \) was calculated using linear regression analysis of the test results, and a value of \( 1.41 \times 10^{-6} \) was obtained for Leadline in comparison to values ranging from \( 2.55 \times 10^{-6} \) to \( 4.50 \times 10^{-6} \) for the different combinations of prestressed and non-prestressed steel. The calculated and the measured crack widths, using this method, were within 40% accuracy.
Where,
\[ w_{\text{max}} = kf_s d_c \sqrt{\frac{A_t}{A_s}} \]  \hspace{1cm} (12)

Where,  
\( w_{\text{max}} \) = maximum crack width at the beam soffit  
\( k \) = bond factor  
\( d_c \) = concrete cover measured from centre of reinforcement

**Method #2: The CEB-FIP Code (1978)**[14] The CEB-FIP Code expression for calculating the crack width using the average spacing between cracks is given by equation (13). The average crack width, \( w_m \), at the beam soffit is estimated using the strain of the prestressing reinforcement after decompression and the crack spacing as given by equation (14). To evaluate the bond factor "\( k_1 \)" in equation (13), two values, 0.4 and 0.8, were selected as initial values for the bond factor. The error of the calculated crack width was interpolated and a value of 0.28 was estimated for the bond factor in comparison to a value of 0.1 for beams prestressed by steel strands. The average crack width of the tested beams was predicted and found to be \( \pm 40\% \) of the measured crack width.

\[ S_{\text{rm}} = 2 \left( c + \frac{s}{10} \right) + k_1 k_2 \frac{d_p}{\rho_r} \]  \hspace{1cm} (13)

\[ w_m = S_{\text{rm}} \zeta \varepsilon_{s2} \]  \hspace{1cm} (14)

Where,
\[ \zeta = 1 - \beta_1 \beta_2 \left( \frac{M_{cr} - M_{dc}}{M_s - M_{dc}} \right)^2 \geq 0.4 \]  \hspace{1cm} (15)

\( \beta \) = \( \beta_1 \beta_2 \), where \( \beta_1 \) is a bond factor, which is calculated for Leadline based on the measured deflection and found to be equal to 1.0, and \( \beta_2 \) is a factor depends on type of loading

\( S_{\text{rm}} \) = average spacing between cracks  
\( c \) = concrete cover  
\( s \) = spacing between longitudinal reinforcement  
\( d_p \) = bar diameter  
\( k_1 \) = coefficient, depends on bond of reinforcement  
\( k_2 \) = coefficient, depends on the shape of the strain diagram  
\( d_p \) = 0.25 \( [(\varepsilon_1 + \varepsilon_2) / 2\varepsilon_1] \), where \( \varepsilon_1 \) and \( \varepsilon_2 \) are strain values in the cracked state at the top and bottom of the zone considered, respectively.  
\( \rho_r \) = \( A_s / A_{\text{cef}} \), where \( A_{\text{cef}} \) is the effective area of concrete  
\( \varepsilon_{s2} \) = strain in the prestressing reinforcement after decompression
**Method #3: Gergely and Lutz [15]:** Gergely and Lutz (1966) recommended the use of equation (16) for the calculation of the maximum crack width at the beam soffit, for reinforced concrete beams. The same equation is proposed for concrete beams partially prestressed by steel reinforcement by the CPCI Design Manual [16]. The bond factor $R$ was modified by Suri and Dilger [13]. The factor $R$, which was linearly interpolated, has a value of $12.5 \times 10^{-6}$ for beams prestressed by Leadline in comparison to a value ranging from $13.7 \times 10^{-6}$ to $25 \times 10^{-6}$ for steel [13]. The predicted crack width for the tested beams, using equation (16) is compared to the measured values and an acceptable agreement within ± 40% was obtained.

\[
w = R \frac{h_2}{h_1} f_s \sqrt[3]{d_c A}
\]

Where,

- $h_1$ = distance from centroid of tensile reinforcement to neutral axis
- $h_2$ = distance from extreme tensile fibre to neutral axis
- $A$ = average effective area around one reinforcing bar

The error of the predicted crack width was compared for the different methods in Table 2. It can be seen that the predicted crack width is within a large margin of error and the least standard deviation of the error was 15% for Gergely and Lutz method. Therefore, it is essential to verify the proposed values for the bond factors for other beams with wide range of span-to-depth ratio and different configurations.

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of data</th>
<th>Mean error (%)</th>
<th>Standard deviation</th>
<th>Minimum error</th>
<th>Maximum error</th>
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<td>Suri and Dilger [13]</td>
<td>42</td>
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<td>Gergely and Lutz [15]</td>
<td>42</td>
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<td>15</td>
<td>-19</td>
<td>42</td>
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</table>

**Conclusions**

This paper describes the serviceability limit states behaviour of concrete beams prestressed by Leadline. The findings of this study can be summarized in the following:

1. Deflection could be accurately predicted by including the concrete tension
2. Simplified $I_e$ method is quite adequate for preliminary design purposes.

3. Crack width can be predicted within 40% accuracy using equivalent bond factors proposed in this study.

References


