DEFLECTION CONTROL OF CONCRETE BEAMS PRESTRESSED BY CFRP REINFORCEMENTS

By Amr A. Abdelrahman and Sami H. Rizkalla

ABSTRACT: Use of carbon fiber reinforced polymers (CFRP) reinforcement for prestressing concrete structures introduces a promising solution for deterioration of concrete structures due to corrosion of steel reinforcements. Due to the low elastic modulus and limited strain at failure of CFRP reinforcement, partial prestressing could be the most appropriate approach to enhance deformability and reduce the cost in comparison to fully prestressed concrete structures. For members reinforced or prestressed with fiber reinforced polymers reinforcements, serviceability requirements may be the governing criteria for the design; therefore, deflection under service loading conditions should be well defined. This paper introduces simplified methods to calculate the deflection of beams prestressed by CFRP reinforcement under short-term and repeated loading. It also examines the applicability of current approaches available to calculate the deflection. Based on an experimental program undertaken at the University of Manitoba, bond factors are introduced to account for tension stiffening of concrete beams prestressed by CFRP. A procedure to determine the location of the centroidal axis of cracked prestressed sections is also proposed. The proposed methods for deflection calculation are calibrated using the results obtained from different experimental programs. Design guidelines are proposed to predict the deflection of beams partially prestressed by CFRP reinforcement.

INTRODUCTION

Use of carbon fiber reinforced polymers (CFRP) reinforcement for prestressing provides an alternative solution to eliminate deterioration of concrete structures due to corrosion of steel reinforcement. The advantages of using CFRP reinforcement, in addition to the noncorrosive characteristics, are the high strength-to-weight ratio, low relaxation, and superior fatigue characteristics (Erki and Rizkalla 1993). During the last decade, several demonstration and prototype structures were built in Canada, Japan, Europe, and the United States (Minosaku 1992; Rizkalla and Tadros 1994; Abdelrahman et al. 1995) in spite of the fact that there is no design code available to design members prestressed by fiber reinforced polymers reinforcement. The current design practice used for several projects of fully prestressed beams requires that the jacking stresses of CFRP reinforcement should not exceed 60% of the tensile strength and the service load is less than the cracking load (Bakht et al. 1996; Recommendation 1997). Partial prestressing using CFRP is introduced in this paper. CFRP reinforcement has excellent fatigue characteristics as it survived 2,000,000 cycles when used in prestressed concrete beams and showed no reduction of the tensile capacity (Abdelrahman et al. 1995). Due to the limited strain at failure of CFRP, partial prestressing improves the deformability of concrete beams and reduces the cost in comparison to fully prestressed beams. Partial prestressing can be introduced either by adding nonprestressed reinforcement or by lowering the jacking stresses in the reinforcement. Due to the high cost of CFRP reinforcement, partial prestressing could be achieved by lowering the jacking stresses to 50% of the tensile strength instead of adding nonprestressed CFRP reinforcement. Consequently, the eccentricity of the prestressing reinforcement can be increased due to the lower prestressing force; hence, the induced stresses in the concrete at the transfer stage will be lower. This increase in the reinforcement eccentricity will result in an increase in the moment capacity of the concrete section. It is also suggested that the increase of the stresses in the reinforcement at the crack location is controlled by the limit stresses under service loading conditions to 60% of the tensile strength of the reinforcement. However, allowing the prestressed beams to crack under service load requires an accurate calculation of the deflection of cracked beams and guidelines to specify the allowable deflection under full service load.

This paper introduces two simplified methods to calculate the deflection of concrete beams partially prestressed by CFRP under short-term and repeated loading. This paper also examines the applicability of the American Concrete Institute (ACI) and the European approaches to calculate the deflection of prestressed beams. Contribution of the concrete in tension, known as tension stiffening, was accounted for by addressing the proper bond factors of CFRP reinforcement. The bond factors were evaluated based on an experimental study undertaken to examine the serviceability of concrete beams partially prestressed by CFRP. This paper also presents a mathematical formula to calculate the location of the effective centroid of cracked sections prestressed by CFRP. The empirical formula is based on the measured strains along the cross section of the tested beams. The effective centroid was used to predict the deflection of prestressed beams with CFRP.

EXPERIMENTAL WORK

Eight concrete beams, 6.2 m long and 330 mm deep, prestressed by CFRP reinforcement, commercially known as Leadline bars produced by Mitsubishi Kasei, Japan, and two beams prestressed by conventional steel strands were tested.

FIG. 1. Cross Section of Tested Beams

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The cross section of the tested beams was a T-section with two different flange widths, 200 and 600 mm, as shown in Fig. 1. The two jacking stress levels used in the study were 50 and 70% of the ultimate guaranteed strength, \( f_y \) of the CFRP specified by the manufacturing company. The level of prestressing and consequently the concrete stress distribution along the section was varied by using two or four bars. The distribution of the Leadline bars in the tension zone was also varied to study its effect on the cracking behavior of the concrete beams prestressed by CFRP bars.

The beams were simply supported with a 5.8-m span and 200-mm projection from each end as shown in Fig. 2. The beams were tested using two quasi-static concentrated loads, 1.0 m apart and cycled 3 times between a lower load level equivalent to 80% of the cracking load of the beam and an upper load level equivalent to 60% of the predicted strength of the beams that is in the range of 1.5–2 times the cracking load depending on the prestressing level. Values of the prestressing force as well as the main characteristics of the tested beams are given in Table 1. Details of the experimental program can be found in Abdelrahman and Rizkalla (1997).

**TABLE 1.** Beam Characteristics

<table>
<thead>
<tr>
<th>Beam number (1)</th>
<th>Type of prestressing tendons (2)</th>
<th>Flange width (3)</th>
<th>Jacking to guaranteed force (%) (4)</th>
<th>Number of prestressing tendons (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CFRP</td>
<td>600</td>
<td>50</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>CFRP</td>
<td>600</td>
<td>50</td>
<td>4-H</td>
</tr>
<tr>
<td>3</td>
<td>CFRP</td>
<td>600</td>
<td>50</td>
<td>4-V</td>
</tr>
<tr>
<td>4</td>
<td>CFRP</td>
<td>600</td>
<td>70</td>
<td>4-H</td>
</tr>
<tr>
<td>5</td>
<td>CFRP</td>
<td>200</td>
<td>50</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>CFRP</td>
<td>200</td>
<td>50</td>
<td>4-H</td>
</tr>
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<td>200</td>
<td>4-V</td>
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<td>8</td>
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<td>2</td>
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<tr>
<td>10</td>
<td>Steel</td>
<td>200</td>
<td>50</td>
<td>2</td>
</tr>
</tbody>
</table>

**FIG. 2.** Test Setup

**SHORT-TERM DEFLECTION**

Short-term deflection of prestressed concrete beams \( \Delta \) can be calculated using an effective moment of inertia \( I_e \) recommended by the ACI Committee 435 (ACI 1995). Deflection can also be calculated using integration of the curvature of sections that can be performed numerically at many or specific sections along the span (Chali 1993).

Before cracking, deflection calculation is based on the properties of the uncracked section. After cracking, calculation of the deflection becomes more ambiguous not only due to the change of the stiffness of the beam but also due to the change of the location of the prestressing force within the cross section.

**Effective Centroidal Distance**

Before cracking, the prestressing force is located at an eccentricity with respect to the centroid of the uncracked transformed section. After cracking and due to tension stiffening, the prestressing force is located at an eccentricity with respect to an effective centroid that is located between the centroid of the transformed cracked and uncracked sections.

Based on the mean measured strains at the sections within the constant moment zone, the depth of the neutral axis was calculated. The centroid of the effective transformed section was calculated at different load levels based on the calculated neutral axis depth and consequently the effective centroidal distance \( y \), was calculated. Based on the experimental program, \( y_e \), which is the distance between the effective centroid of the section and the extreme compression fiber, can be determined for beams prestressed by CFRP for a given service moment \( M_s \) as proposed by (1). The expression is given in terms of the cracked and uncracked centroidal distances \( y_c \) and \( y_u \), respectively, and takes into consideration the decompression moment \( M_{dc} \) due to prestressing force. In this study, the decompression moment is defined as the moment that produces zero stress at the extreme concrete fiber of the precompressed zone.
The measured values are compared in Fig. 3 to the cracked and uncracked centroidal distances \( y_c \) and \( y_s \) as well as to the proposed expression for \( y_s \) given by (1). It should be mentioned that based on Branson and Trost (1982), the effective centroidal distance can be predicted for beams prestressed by steel using (1) with a power equal to 2.5. This suggests that \( y_s \) for beams partially prestressed with CFRP bars is larger than that for beams prestressed by steel subjected to the same prestressing and loading conditions. This behavior could be attributed to the larger crack spacing; hence, larger uncracked portions are observed for beams prestressed by CFRP

\[
y_s = \psi y_s + (1 - \psi) y_c \leq y_s
\]  
(1)

where

\[
\psi = \left( \frac{M_c - M_{nc}}{M_s - M_{nc}} \right), \quad M_s > M_{cr}
\]  
(2)

Proposed Simplified Method

Test results indicate that deflection of beams prestressed by CFRP bars can be determined using the effective moment of inertia \( I_s \) proposed by ACI Committee 435 (ACI 1995) and given by (3)

\[
I_s = \psi y_s + (1 - \psi) y_c \leq I_s
\]  
(3)

The proposed expression to calculate the deflection for beams prestressed by CFRP is given as follows:

\[
\Delta = -k_p \frac{\psi_d (d_p - y_s) k_i}{E_i I_s} + k \frac{M_i k_i}{E_i I_s}  
\]  
(4)

The expression in (4) accounts for the deflection due to prestressing force \( P \), and the deflection due to service moment \( M_s \) due to dead and live loads. It also accounts for the eccentricity of the prestressing force after cracking \( (d_p - y_s) \). The values of \( y_c \) and \( y_s \) are given by (1) and (2), respectively. The factors \( k_p \) and \( k \) depend on the shape of the prestressing cables and the loading pattern.

Deflection due to specified live load should be calculated as the difference between the deflection due to the total service load and the deflection due to dead load. This is not only due to the change of the effective moment of inertia \( I_s \) but also due to the change of the eccentricity of the prestressing force after cracking that is given in terms of \( y_s \) in (4). For uniformly distributed load and straight prestressing tendons, the value of \( k_p \) and \( k \) are 1/8 and 5/48, respectively.

The deflection of the tested beams was predicted and shown in Fig. 4 for a typical beam prestressed by CFRP and tested in the experimental program. Comparing the measured and the computed deflection, it was found that 82% of the calculated deflection is within \( \pm 20\% \) of the measured values.

Deflection using Curvature Approach

The European approach suggests calculation of the deflection based on curvature integration at many sections along the span. Because formation of the cracks in concrete beams occurs randomly, tension stiffening is typically accounted for using empirical equations. Tension stiffening depends mainly on the bond characteristics of the reinforcement that, in turn, depend on many factors including the type of reinforcement, elastic modulus, and Poisson's ratio. Limited attempts have been made to evaluate the tension stiffening of beams with FRP reinforcement (GangaRao and Faza 1991; Bennomkrame et al. 1996).

The CEB-FIP Code 1990 (CEB-FIP 1993) interpolates the effective curvature \( \phi \), between the curvature of the gross and the cracked sections \( \phi_g \) and \( \phi_{cr} \), respectively, as given by (5). The interpolation factor \( \xi \) is defined in (6). The factor \( \beta_1 \) refers to the bond condition of the reinforcement and the factor \( \beta_2 \) refers to the type of the applied load

\[
\phi_{cr} = (1 - \xi) \phi_g + \xi \phi_{cr}
\]  
(5)

\[
\xi = 1 - \beta_1 \beta_2 \left( \frac{M_{cr} - M_{nc}}{M_s - M_{nc}} \right)^2 \geq 0.4 \quad \text{and} \quad M_s > M_{cr}
\]  
(6)
FIG. 5. Predicted Deflection of Beams Prestressed by CFRP Bars Using CEB-FIP Code

FIG. 6. Strain and Stress Distribution of Concrete Section Prestressed by CFRP Bars

FIG. 7. Predicted Deflection of Beam Prestressed by CFRP Bars Using Strain Compatibility
Using the test results of the experimental program, the value of $\beta_1$ was found to be equal to 1.0 for CFRP bars. It should be noted that $\beta_1$ is equal to 1.0 for high bond steel bars and 0.5 for smooth steel bars (CEB Manual 1985). Using a value of 1.0 for $\beta_1$, the estimated deflection of the tested beams was in good agreement with the measured values, as shown in Fig. 5 for two beams with different cross sections. This is due to the deformed surface of the CFRP reinforcement used in this study, which increased the bond with the concrete. The value of $\beta_2$ is 1.0 for short-term loading and 0.5 for sustained or cyclic loading.

The effective curvature was also calculated based on the slope of the strain profile of cracked sections. Strain compatibility and equilibrium of the internal forces including the tensile force in the concrete were employed to calculate the deflection of the beams. Eq. (7), proposed by Collins and Mitchell (1991), was used to calculate the tensile stresses in the concrete after cracking. The factor $\alpha_1$ depends on the bond characteristics of the reinforcement, whereas $\alpha_2$ depends on the type of loading and is taken as 1.0 for short-term loading and 0.7 for sustained and/or repeated loading

$$f_{ot} = \frac{\alpha_1 \alpha_2 f_c}{1 + \sqrt{500\varepsilon_t}} \quad \varepsilon_t > \varepsilon_r$$

Equation (7) was used to calculate the tensile stresses in the concrete with two locations on the tension side of the beam: (1) Below the neutral axis where the tensile stresses increase linearly up to the rupture strength; and (2) on the effective embedment zone around the reinforcement, as shown in Fig. 6. The deflection was calculated using integration of the effective curvature at several sections along the beam. A very good agreement between the predicted and the measured values was obtained as shown in Fig. 7 for one of the tested beams. Applying the curvature integration at selected sections overestimates the deflection beyond the service load range as shown in Fig. 7. The deflection of a beam prestressed by CFRP bars with an 8.8-m span tested in a different research program by Fam et al. (1995) was predicted using a value of 0.6 for $\alpha_1$ and compared with the measured deflection in Fig. 8.

Comparing the measured and the computed deflection using either the CEB-FIP Code or the strain profile to calculate the curvature, it was found that 93% of the calculated deflection is within ±20% of the measured values. Ignoring the tension stiffening significantly reduces the accuracy of the deflection.
prediction to 56% as shown in Fig. 9. Both the computed and the measured deflections can be found in Abdelrahman (1995).

**DEFLECTION UNDER REPEATED LOADING**

Because partially prestressed beams are designed to crack under superimposed dead and live loads, the behavior of the beams under service load cannot be considered elastic. It is required to calculate the deflection due to repeated loading for beams subjected accidentally to loads higher than the specified service load. Deflection may be also calculated under the effect of a fraction of the live load representing the sustained live load.

The behavior of the tested concrete beams partially prestressed by CFRP before cracking was elastic and linear. After cracking, the load-deflection behavior under repeated loading is not perfectly elastic despite the fact that CFRP reinforcement is elastic up to failure. During unloading of the beams, the deflection does not follow the same path as the initial cycle. This is attributed to cracking of the beams and the non-recoverable creep of concrete. The measured deflections of the beams in the second and third cycles were found to be identical and higher than that in the initial cycle. The deflections of the beams, after reloading to the same load level, follow the same path as in the initial cycle. This behavior is typical for beams prestressed by CFRP bars and steel strands as shown in Fig. 10.

**PROPOSED MODEL**

The typical behavior of concrete beams partially prestressed by CFRP bars under repeated loading is shown in Fig. 11. To illustrate this behavior, the loading path of a typical beam is shown from point O, which represents zero load, to cracking point A and point B, which is higher than the cracking load. The slope of the line OB is proportional not only to the effective moment of inertia $I_e$ but also to the eccentricity of the prestressing force calculated at point B. If the beam is unloaded at point B and the behavior of the beam is perfectly elastic, the beam will follow the path BAO, which is the same path as the initial loading, and the total deformation of the beam should be completely recovered. Conversely, if the behavior of the beam is perfectly inelastic, the beam will follow the path BE, which is parallel to the line OA, and the beam will have a residual deformation corresponding to the deflection OE. The actual behavior of the beam is between these two cases and is represented by the path BCD. In spite of the fact that the CFRP reinforcement and the concrete are linear at this service level, the beam will have a residual deformation $\Delta_r$, equivalent to OD. This residual deformation is proportional to the load at which the beam is unloaded.

The bilinear behavior at unloading and reloading of the prestressed beams shown in Fig. 11 was observed for all the tested beams including the beams prestressed by steel strands. The load-deflection relationship is considered to be linear from point D, up to a load $P_{de}$ causing the decompression moment $M_{de}$ defined by point C. The deflection within the range DC can be estimated using the uncracked moment of inertia $I_c$. The deflection is also assumed to be linear between point C and point B at which the beam is unloaded. The stiffness of the beam at point B, which is proportional to the slope of the line DB, is based on a moment of inertia $I_{rep}$, which is higher than the inertia $I_c$ at the same point. The deflection at any point...
The effective centroidal distance $y_e$ was found to be smaller under repeated load than that for the initial loading; however at a load level of $P_{rep}$, $y_e$ has the small value for the initial and subsequent cycles as shown in Fig. 12. Therefore, (1), which is used to calculate $y_e$ at initial loading, can be applied to estimate $y_e$ at the unloading level $P_{rep}$.

The proposed model was applied to the tested beams and found to be in good agreement with the measured deflection. The average value and the standard deviation of the difference between the predicted and the measured deflections were 1 and 11%, respectively. A predicted load-deflection relationship for one of the tested beams prestressed by CFRP bars under repeated loading is shown in Fig. 13. The beam was unloaded at two load levels, at the service load and at 70% of the failure load. Both the measured and the predicted deflection are in a good agreement.

CONCLUSIONS

A simplified method was proposed to calculate the deflection of concrete beams partially prestressed by CFRP bars using the $I_e$ approach. An expression was introduced to estimate the location of the effective centroid of concrete beams prestressed by CFRP bars. The method was calibrated using available data of large-scale beams with CFRP bars. Empirical bond values were proposed to account for the tension stiffening of beams prestressed by CFRP bars using
tion of the deflection using integration of the curvature along
the beam span or using the proposed simplified method was
found to be in good agreement with the measured deflection.

Behavior of concrete beams prestressed by CFRP under re-
peated load is similar to that for beams prestressed by steel
provided that the steel is in the elastic stage before unloading.
A model was proposed to calculate the deflection of beams
prestressed by CFRP bars under repeated loads. The proposed
model was compared with available data, and a very good
agreement was obtained between the predicted and the mea-
sured values.

APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

- $A_p$: area of prestressing FRP;
- $d_p$: depth of prestressing reinforcement;
- $E_c$: elastic modulus of concrete;
- $f_y$: tensile strength of concrete;
- $f_c$: rupture strength of concrete;
- $I_{cr}$: cracked moment of inertia;
- $I_e$: effective moment of inertia;
- $L$: span of beam;
- $M_{cr}$: cracking moment;
- $M_{de}$: decompression moment;
- $M_{np}$: moment due to repeated load;
- $M_s$: service moment;
- $n_r$: modular ratio = elastic modulus of FRP/elastic modulus
  of concrete;
- $P_{de}$: decompression load;
- $P_e$: effective prestressing force;
- $P_{max}$: maximum repeated load;
- $y_{cc}$: distance between centroid of section and compression fi-
  ber based on cracked section properties;
- $y_c$: distance between centroid of section and compression fi-
  ber accounting for tension stiffening, calculated at one
  section or along beam;
- $y_s$: distance between centroid of section and compression fi-
  ber based on gross section properties;
- $\alpha_p$: factor depends on bond condition of reinforcement;
- $\beta_p$: factor depends on type of applied load;
- $\Delta$: deflection of beam;
- $e_s$: tensile strain of concrete;
- $\phi_c$: curvature of section based on cracked section analysis;
- $\phi_s$: curvature of section accounting for tension stiffening;
- $\phi_{ns}$: curvature of section based on gross section analysis.

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