Deflection of Concrete Slabs Reinforced with Advanced Composite Materials

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ABSTRACT

Fibre Reinforced Plastic (FRP) bars are currently used for special concrete structures in areas sensitive to magnetic fields and severe environmental conditions which accelerate corrosion of the steel reinforcements, and consequently deterioration of the structure. This paper presents analysis of the experimental results obtained by testing eight one-way concrete slabs reinforced with glass-fibre reinforced plastic (GFRP) rebars, carbon-fibre reinforced plastic (CFRP) rebars and conventional steel reinforcement. The slabs were tested under four-point static loading to investigate their flexural limit states, including the behaviour prior to cracking, at cracking, ultimate capacities and modes of failure. In this paper, the deflection of concrete slabs reinforced with FRP rebars is investigated. Based on this investigation, design recommendations and guidelines are proposed for the deflection and cracking load calculations.
INTRODUCTION

Deterioration of conventional steel reinforcement due to corrosion reduces the service life of bridge decks and parking garages. Significant fluctuation of the temperature and use of salt for deicing accelerate the corrosion process of the steel reinforcements. In Canada, it is estimated that the cost of repair of parking structures is in the range of four to six billion dollars. The estimated repair cost for existing highway bridges in U.S. is over 50 billion dollars, and one to three trillion dollars for all concrete structures (Bedard, 1992). Excessive corrosion problems also exist in Arabian Gulf countries (Makhtouf et al., 1991). The exterior of reinforced concrete structures in these countries are subjected to an extremely aggressive environment due to high temperatures and humidities.

Use of non-corrosive fibre reinforced plastic, FRP, rebars in place of steel reinforcement has been investigated as an alternative to overcome the corrosion problem in bridge decks, parking garages, water and wastewater treatment facilities, marine structures and chemical plants. In addition to their excellent non-corrosive characteristics, FRP reinforcements have high strength-to-weight ratio and good fatigue properties. However the linear elastic behaviour of these materials in addition to their low modulus of elasticity require special design considerations.

In this paper the flexural behaviour of concrete slabs reinforced with FRP reinforcements is investigated. Analytical methods are proposed, based on the experimental program conducted at the University of Manitoba (Michaluk et al., 1992), to predict deflections and cracking of such slabs.

MATERIAL CHARACTERISTICS

Although GFRP bars possess the lowest tensile strength in comparison to other available FRP reinforcements, they have the advantage of being the least expensive (Bedard, 1992), along with their non-corrosive, magnetically neutral and high strength to weight ratio characteristics (Challal and Bennokrane, 1993). Therefore, GFRP reinforcements are an excellent candidate for reinforced concrete structures subjected to aggressive environmental conditions and for those sensitive to magnetic fields. The rupture strength, \( f_{ru} \), and the elastic modulus, \( E_f \), of the FRP reinforcements used in this program are shown in Table 1.

The GFRP bars are manufactured by pultration of E-glass continuous fibres and thermosetting polyester resin. To enhance the bond characteristics, the surface is wrapped by helically glass fibre strands and covered by a mixture of a known grain size of sand and polyester resin. The CFRP reinforcement used in this test program is an 8 mm diameter rod produced by Mitsubishi Kasei, Japan. The CFRP rods are fabricated using continuous coal tar pitch-based continuous fibre and epoxy resin (Mitsubishi Kasei, 1992). The measured average compressive strengths of the concrete used for the slabs ranged from 60.0 MPa to 66.3 MPa at the time of testing, with a maximum aggregate size of 14 mm.

EXPERIMENTAL PROGRAM

Details of the eight prototype one-way concrete slabs, reinforced by three different reinforcement materials, tested in this program are given in Table 1. Five specimens were reinforced by GFRP bars, two specimens were reinforced by conventional steel rebars and one specimen was reinforced by CFRP rods. The three slabs reinforced by CFRP and steel reinforcements are used as control specimens to compare the behaviour of the slabs reinforced by GFRP bars.

The length and width of all the slabs were 3500 mm and 1000 mm, respectively, with a clear span of 3000 mm, which were kept constant throughout the study. The thickness of slabs was 150 mm according to the requirements of the Canadian Design Code (CAN3-A23.3-M94, 1994) and the program was expanded to include 200 mm thick slabs. A concrete cover of 38 mm was used for the longitudinal reinforcements. Initially, the slabs were designed to achieve the classical three modes of failure, including: rupture of the reinforcements; simultaneous rupture of the reinforcements and crushing of the concrete and; crushing of the concrete while the reinforcement remains elastic. This was accomplished by using reinforcement ratios less, equal and more than the balanced reinforcement ratio, \( p_b \), for the slabs, respectively. The slabs were instrumented to measure the applied load, midspan deflections, strains in the extreme compression fibres of the concrete, strains in the reinforcements, strains in the
concrete at the level of reinforcements, crack widths within the constant moment zone and bond slippage at both ends of the specimens.

**Cracking of FRP Reinforced Concrete Slabs:**

Many researchers have found that the cracking of non-prestressed concrete structures reinforced with FRP bars is one of the main criteria that would govern the design of such structures. The nature of the crack pattern, propagation and height in FRP reinforced concrete structures was found to be different from that in steel reinforced concrete structures. This is due to the linear stress-strain diagram, different bond characteristics and lower modulus of elasticity of FRPs relative to the steel reinforcement.

Due to the relatively low modulus of elasticity of the FRP rebar, the transition from the uncracked section to the fully cracked section is more abrupt in FRP reinforced concrete slabs than in steel reinforced concrete slabs. During the experimental program, the immediate crack widths in slabs reinforced with FRP bars were larger than those in slabs reinforced with conventional steel. At the cracked sections, slippage of the GFRP rebars was observed due to the debonding of the outer coating of the rebar and the inner core.

**Cracking Load \( (P_{cr}) \):**

The cracking moments of the test slabs were in the first instance estimated based on the section modulus of the transformed cross section, Eq. [1]:

\[
M_{cr} = f_r \frac{I_r}{y_t}
\]

where \( M_{cr} \) is the cracking moment; \( f_r \) is the modulus of rupture; \( y_t \) is the distance from the centroidal axis of the transformed cross section to the extreme fibre in tension; and \( I_r \) is the transformed moment of inertia of the cross section. The application of Eq.[1] to the test-slabs resulted in a much higher cracking load than the experimental cracking load as shown in Fig. 1. The predicted cracking loads according to Eq.[1] were 15% to 65% higher than the actual cracking loads from the tests. Due to the debonding taking place between the FRP rebar and the surrounding concrete, it is suggested that the effective modulus of a slab cross section is less than the gross or transformed modulus. The full concrete cover at the rebar location is not contributing to the cross section moment of inertia. Therefore the effective depth of the concrete section lies between \( t \), the total depth of the section, and \( (t-c) \) where \( c \) is the clear concrete cover. The effective depth for the moment of inertia calculations was taken as \( (t-c/2) \) in this study.

The cracking moment was recalculated for the test slabs based on the section modulus of the reduced slab thickness as given by Eq. [2]:

\[
M'_{cr} = f_r \frac{2 I_r}{(t - \frac{c}{2})} \quad \text{and} \quad I_r = \frac{b (t - \frac{c}{2})^3}{12}
\]

where \( M'_{cr} \) is the cracking moment based on reduced slab thickness; \( I_r \) is the reduced moment of inertia; \( b \) is the width of the cross section; and \( c \) is the clear concrete cover. This method gives more realistic estimation to the cracking moment and hence the cracking load as shown in Fig. 1.
Deflection of FRP Reinforced Concrete Slabs:

This study examines the applicability of the various methods currently available for estimating the deflection of non-prestressed concrete one-way slabs. Based on an experimental program conducted at the University of Manitoba, modifications were introduced to these methods to account for the nature of crack pattern and propagation in FRP reinforced concrete slabs.

A cracked member behaves, in general, as a member of variable cross section since the rigidity is much reduced at the cracked zone. At a crack there are no tensile stresses in the concrete. However, between the cracks, the concrete in tension contributes to the flexural rigidity of the member resulting in stiffening of that member.

Deflection Prediction Models. Different detailed models are presented to account for the tension stiffening in FRP reinforced concrete slabs. A simple design method for estimating the post-cracking deflection of these slabs is introduced. A comparison between the measured deflections and the predicted values is presented.

ACI and Canadian Building Codes Model. According to the current ACI Building Code (Building, 1989) and the Canadian Building Code (CAN3-A23.3-M94, 1994), the immediate deflection of a cracked member can be estimated using constant effective moment of inertia $I_e$, given by the empirical equation:

$$I_e = \left( \frac{M_{cr}}{M_a} \right)^3 I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \leq I_g$$

where $M_a =$ maximum applied moment in the member at a stage deflection is calculated; $I_{cr} =$ moment of inertia of a cracked section transformed to concrete, and $I_g =$ moment of inertia of gross concrete section. Deflections estimated according to Eq. [3] are compared to the experimental results in Fig. 2. Since the cracking moment calculated from Eq. [1] was much higher than the experimental cracking moment, deflections were modified based on the cracking moment proposed in Eq. [2], $M'_{cr}$. The deflection estimated according to the modified ACI method considering the reduced cracking moment is shown also in Fig. 2. It can be seen from Fig. 2 that the current ACI model greatly underestimates the deflection of FRP reinforced concrete slabs.

CEB-FIP Code (1990) Model. Deflection of concrete members is primarily estimated using integration of the curvature along the span of the slab. The curvature, $\Phi$, of any section can be calculated using the applied moment, $M$, and the corresponding rigidity of the section, $E_c I$, as given by Eq. [4] where $E_c$ is the elastic modulus of the concrete and $I$ is the moment of inertia of the section.

$$\Phi = \frac{M}{E_c I}$$

The deflection at the centre of the simply-supported slab can be calculated by the geometric relationship:

$$\Delta = \int_0^{\nu/2} \Phi \times dx$$

[5]
where \( x \) = distance measured from the left end; and \( l \) = span of the slab. The integration in Eq. [5] can be performed numerically at many sections along the span as given by Eq. [6]:

\[
\Delta = \int_0^{l/2} \Phi x \, dx = \sum_{i=0}^{l/2} \frac{\Phi_i \, x_i + \Phi_{i+1} \, x_{i+1}}{2} \Delta x_i
\]

where \( \Phi_i \) and \( x_i \) are the curvature and the distance at the \( i \)th section, respectively and \( \Delta x_i \) is the incremental distance between the \( i \)th and \( i+1 \) sections. Increasing the number of sections will increase the accuracy in case of non-linear variation of the curvature (Ghali, 1993).

Curvature at a cracked section is substantially greater than the curvature at a non-cracked section subjected to the same moment. At a cracked section, the concrete in tension is ignored commonly in the calculation of the curvature. The curvature in this stage is referred to as \( \Phi_\alpha = M/EI_\alpha \). The curvature of a non-cracked section will be referred to as \( \Phi_g = M/EI_g \). Within the spacing between cracks, the concrete in tension contributes to the stiffness and reduces the curvature and, hence, the deflection. To account for the tension stiffness effect, a mean curvature value \( \Phi_m \) is used in deflection calculation. The value of \( \Phi_m \) is estimated according to the CEB-FIP Code by introducing an interpolation empirical factor \( \zeta \) between the curvature of the gross and the cracked sections, \( \Phi_g \) and \( \Phi_\alpha \), respectively as given by Eq. [7]:

\[
\Phi_m = (1 - \zeta) \Phi_g + \zeta \Phi_{cr}
\]

\[
\zeta = 1 - \beta' \left( \frac{M_{cr}}{M} \right) \geq 0.4 \quad \text{and} \quad M \geq M_{cr}
\]

where \( M \) is the bending moment at the section considered; \( \beta' = \beta_1 \beta_2 \), where \( \beta_1 \) is a factor depends on the bond conditions of the reinforcement, \( \beta_1 = 1 \) for high bond bars; and \( \beta_2 = 1 \) and 0.5, respectively, for first loading and for loads applied in a sustained manner or for a large number of cycles. For most practical applications, the factor \( \beta' \) is taken as 0.5 (Ghali, 1993). This value of \( \beta' \) is used in the application of Eq. [8] in this paper. It has to be noted that \( \Phi_m \) represents the curvature at one section, not an average curvature for a member. Also when the deflection of a member is evaluated using constant effective moment of inertia, \( I_e \), given by Eq. [3], the flexural rigidity \( EI \) is considered dependent on the maximum bending moment applied to the member, \( M_{cr} \). In the CEB-FIP (1990) model the curvature and hence the deflection depends on the moment at each section considered.

The deflection of the test-slabs was estimated based on integration of the mean curvature at several sections along the span. The value of the cracking moment was reduced as suggested before in Eq. [2]. Comparison of the deflections shows that the integration method, when modified to account for the reduced cracking moment in the FRP slabs, provides an accurate prediction for the deflection as shown for one of the test-slabs in Fig. 3.

**Simplified Method According to the CEB-FIP Model 1990.** Favre and Charif (1994) introduced a method for the deflection calculations according to the CEB-FIP Model Code (1990) for concrete structures reinforced with conventional steel. According to this method, the deflection can be estimated from Eq. [9]:

\[
\text{Deflection} = \frac{M}{EI} \left( 1 - \frac{2}{9} \frac{M_{cr}}{M} \right)
\]
where $\Delta_1$, $\Delta_2 = \text{deflections in states 1 and 2, respectively for simple bending, where a non-cracked section is said to be in state 1 and a cracked section with the concrete in tension is ignored is said to be in state 2; in state 1 the transformed moment of inertia is used to calculate } \Delta_1; \text{ and in state 2 the cracked moment of inertia is used to calculate } \Delta_2; \text{ } \beta' = \text{cracking moment reduction factor } = \beta_1, \beta_2, \text{ as defined before; and } M = \text{maximum applied moment. It can be noted in Eq. [9] that the expression } [(\Delta_2 - \Delta_1) \cdot \beta' \cdot (M_c / M)] \text{ represents the tension stiffening for concrete structures reinforced with conventional steel.}

Equation [9] can be simplified for the case of FRP reinforced concrete slabs by replacing the transformed moment of inertia used in state 1 by the gross moment of inertia $I_g$, neglecting the inertia of the reinforcement. This is due to the typical low modulus of elasticity of the FRP reinforcements. Also the factor $\beta'$ can be taken equal to 0.5 as was suggested before in the integration method. In order to account for the reduced cracking moment of FRP reinforced concrete slabs, the cracking moment $M'_{cr}$ will be used in this method. Therefore, Eq. [9] can be rewritten in the following form:

$$\Delta = K \frac{L^2}{E_c} \left( \frac{0.5 \cdot M'_{cr}}{I_g} + \frac{1}{0.85} \cdot \frac{M - 0.5 \cdot M'_{cr}}{I_{cr}} \right)$$

[10]

where $K$ is a factor depends on the loading and boundary conditions of the slab, $K=23/216$ for the conditions of the test slabs; and $L$ is the span. To account for the reduction in tension stiffening in fully cracked FRP concrete section, the second term in the right hand side of Eq. [10] was multiplied by a factor of 1/0.85. Such factor was introduced before by the European Concrete Committee (CEB), (Branson, 1977) to reduce the slope of the load deflection curve after cracking.

The deflections of the test slabs were estimated using Eq. [10] and compared to the measured values. Good agreement was found between the experimental and the theoretical results as shown in Fig. 4. It should be noted that the deflection according to Eq. [10] depends on the gross and the cracking moments of inertia of the slab which results in simple and quick calculations.

**SUMMARY AND CONCLUSIONS**

Measured deflections and crack widths of eight simply supported FRP reinforced concrete slabs were used to evaluate the different methods available to examine serviceability of such slabs. Good agreement was obtained between results from the proposed methods of analysis and the experimental results.

Cracking loads of FRP reinforced concrete slabs are less than those of steel reinforced concrete slabs due to the debonding that takes place between the concrete and the FRP rebars. A method for predicting the cracking load of FRP reinforced concrete slabs is introduced.

Integration of the curvature along the slab span, taking into account the reduced cracking moment of the FRP reinforced concrete slabs was found to be the most accurate method to predict the deflections of such slabs.

A simplified and quick method was introduced to estimate the deflections of FRP reinforced concrete slabs. Results of this method can be considered accurate from the engineering practical point of view.

Fibre reinforced plastics should not be seen as a direct replacement for conventional steel, they are new materials with new characteristics. The post-cracking behaviour of FRP reinforced concrete slabs is different from that of steel reinforced concrete slabs.
REFERENCES


Table 1 - Characteristics of the Test-Slabs

<table>
<thead>
<tr>
<th>Slab</th>
<th>Reinforcing Material</th>
<th>Thickness of Slab (mm)</th>
<th>f_v (%)</th>
<th>E Modulus (MPa)</th>
<th>f_c (MPa)</th>
<th>E (GPa)</th>
<th>P_0 (kN)</th>
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<tbody>
<tr>
<td>I-150-A</td>
<td>GFRP</td>
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<td>0.487</td>
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<td>4.73</td>
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<td>6.66</td>
<td>692</td>
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<td>0.955</td>
<td>66</td>
<td>6.23</td>
<td>692</td>
<td>41</td>
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<tr>
<td>S-150-T</td>
<td>Steel</td>
<td>150</td>
<td>0.962</td>
<td>60</td>
<td>7.2</td>
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<td>NA*</td>
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<td>I-200-A</td>
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</tbody>
</table>

* Denotes not applicable
FIG. 1. Cracking Loads of the Test Slabs

FIG. 2. Load-Deflection of Slab I-150-A - ACI Code

FIG. 3. Load-Deflection of Slab I-150-A - Integration Method

FIG. 4. Load-Deflection of Slab I-150-B - Simplified Method