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Abstract

Two extensive and independent experimental programs have been conducted to study the cracking behavior of reinforced concrete members subjected to pure tension in the presence of transverse reinforcement. The first program involved the testing of eighteen reinforced concrete segments and was mainly designed to examine the applicability of the existing equations for predicting crack spacing and widths. The segments were reinforced in two directions and loaded in uniaxial tension beyond the yield stress of the steel. The measured values of the final crack spacing were compared to the values presented by other researchers. Based on this comparison, a simplified and refined expression for prediction of crack spacing is proposed.

The second experimental program involved the testing of nine reinforced concrete segments identical in all parameters, except the spacing of transverse reinforcement. The program was designed to study the influence of transverse reinforcement spacing on crack behavior. A methodology for predicting the crack spacing in reinforced concrete members with variable transverse reinforcement spacing is presented.

Introduction

For many reinforced concrete structures, the problem of tensile cracking is almost unavoidable. However, there are certain types of structures such as tanks, silos, and nuclear reactor containment structures, where tensile cracks can pose serious problems. All such structures are generally reinforced in two directions and mainly subjected to membrane tensile forces. There are many research reports on crack behavior in reinforced concrete members. However, internationally, there is remarkably little agreement on theories or equations for predicting crack width and spacing. In addition, most of the experimental work to date has tended to concentrate on the behavior of members reinforced in one direction only. Very little work is available on crack behaviour of members reinforced in two directions which represent most of the construction details of any structure.

To study the cracking behavior of reinforced concrete members subjected to pure tension in the presence of transverse reinforcement, two extensive and independent experimental programs, sponsored by the Natural Science and Engineering Research Council, have been conducted at the University of Manitoba. The first program involved the testing of eighteen reinforced concrete segments and was mainly designed to examine the applicability of
the existing equations for predicting crack spacings and widths in the presence of transverse reinforcement. The segments were reinforced in two directions and loaded in uniaxial tension beyond the yield stress of the steel. The main parameters varied were the amount of concrete cover (c), concrete thickness (t), and percentage of steel (p), as given in Table 1. For each specimen, strains, crack width and crack spacing were measured during testing. The measured values of the final crack spacing were compared to the values based on equations presented by others [1, 2, 7].

The second experimental program involved the testing of nine reinforced concrete segments identical in all parameters, except the spacing of transverse reinforcement, as given in Table 1. The program was designed to study the influence of transverse reinforcement spacings on crack behaviour. The proposed expression based on the first part of the study was applied and compared with results obtained from the second experimental program.

Test Specimens

All segments in both programs were reinforced in two directions with deformed bars. Longitudinal reinforcement was spaced at 3" centre to centre and extended 11" beyond each end of the specimen as shown in Figure 1. For the first program transverse reinforcement was provided by 10M bars spaced at 3" centre to centre on both faces for all the specimens.

In the second program, the nine segments were divided into three groups of three segments each, so that three different spacings of 10M transverse bars could be tested. The spacings selected were 2", 4", and 6" centre to centre on both faces.

All concrete used in fabrication of the segments was designed for a nominal ultimate strength of 5000 psi and was mixed in the laboratory. For each segment, six standard 6" x 12" cylinders were cast and tested to determine the compressive and tensile strength of concrete. The reinforcement used in the segments consisted of hot rolled deformed bars grade 60. For each specimen, samples of the steel bar were tested for tension to determine yield stress, ultimate stress, and modulus of elasticity. All samples showed typical ductile behavior and well defined yield point. After casting, the specimens were left to dry in open air for about one hour, then placed in a curing room for 7 days. Following this 7-day curing time, the specimens were removed from the curing room and covered with wet burlap and remained in the "air-dry" stage for at least an additional 21 days before testing.

Testing Apparatus and Procedure

The loads were applied by using a 600,000 lb. capacity universal testing machine. Load was transmitted from the loading machine to the specimen by using specially designed end fittings as shown in Figure 2. The end fittings were very rigid in order to achieve uniformity of loads from the machine to each load cell. Each reinforcing bar was connected to a separate specially designed load cell, which was used to adjust the load transferred to each reinforcing bar to provide uniformity of forces applied to the segment. The uniformity of loads in each reinforcing bar was achieved by tightening and loosening the load cells.
After balancing of the loads, testing of the specimen commenced by applying an initial load of 3000 lbs. to ensure proper seating of the segment. This load was considered the "Zero" load condition for the test. The load was then applied in increments of 5000 lbs.

During testing, a continuous record of deformation over a 30" gauge length was made using a linear variable differential transducer (LVDT) as shown in Figure 2. In addition, the number of cracks and their widths were measured at regular intervals with a travelling micrometer-microscope with an accuracy of 0.02 mm as shown in Figure 4. As well, readings for mechanical strain gauges were recorded. The crack patterns were marked and numbered at the end of each increment. Testing was terminated when the load approached the yield point. The specimen was then unloaded and the final crack pattern was noted. The full details of the testing apparatus and procedure was described in Reference 5.

Governing Equations

The mechanism of cracking of axially loaded reinforced concrete members is suggested in many studies to depend on the bond failure between the concrete and steel [1, 6, 7, 10]. As a different approach to the problem, Broms [3, 4] proposed a cracking mechanism based on an elasticity analysis of concrete stresses. He concluded that the crack spacing is directly related to the concrete cover (c).

Goto [6] observed from his tests results that, when the member is subjected to tensile forces, internal cracks will occur behind the ribs of the deformed steel bars as shown in Figure 3, and they are influenced by the diameter of the reinforcement bar (d). After the formation of these internal cracks, the axial tensile force is carried by the uncracked shell of the concrete section. The small concrete teeth will resist the interlocking forces by flexural resistance. As one or two of the concrete teeth lose their resisting strength, almost no bond length \( L_0 \) will extend equally each way from the crack as shown in Figure 3.

Leonhardt [7] showed that when the concrete stress due to the applied load reaches the tensile strength, the concrete starts to crack, causing a sudden jump in steel stress. If this sudden increase in the steel stress is large enough, some bond-slip can occur and the crack formation is likely to be a combination of bond-slip and internal cracking as suggested by Goto [6].

Leonhardt assumes the minimum possible value for the average crack spacing at the final crack pattern \( S_L \) is:

\[
S_L = \frac{1}{2} L_0 + L_t, \quad \text{(Eq. 1)}
\]

where \( L_t \) is the transfer length which represents the length of active bond stress and \( L_0 \) is the length of almost no bond stresses. Based on experimental results, Leonhardt estimated the length \( L_0 \), as:

\[
L_0 = \frac{f_{s_2,cr}}{6500} d \quad (f_{s_2,cr} \text{ in psi}) \quad \text{(Eq. 2)}
\]
where \( d \) is the diameter of the reinforcement and \( f_{S2,cr} \) is the stress in the steel at the crack immediately after cracking. Leonhardt's expression for the transfer length (\( L_t \)) is:

\[
L_t = K_1(a, c) + 0.01 \frac{d}{p}
\]  
(Eq. 3)

where \( p \) is the percentage of steel reinforcement, and \( K_1 \) is a factor depending on the concrete cover of the bars, \( c \), and the spacing between the longitudinal bars, \( a \). The complete derivation is given in Reference 7.

Based on this proposed cracking mechanism, the average crack width \( W_m \) can be computed as follows:

\[
W_m = L_0 \varepsilon_{S2} + L_t \varepsilon_m
\]  
(Eq. 4)

where \( \varepsilon_{S2} \) is the steel strain at the cracked section, \( \varepsilon_m \) is the average gross strain measured over the cracks including concrete contribution within the transfer length and is defined as follows:

\[
\varepsilon_m = \varepsilon_{S2} \left[ 1 - \frac{f_{S2,cr}}{f_{S2}} \right]^{\frac{1}{2}}
\]  
(Eq. 5)

where \( f_{S2} \) is the steel stress at the crack.

Similarly, Beeby [1, 2] suggested an expression for the minimum crack spacing given by:

\[
S_b = 1.33 \, c + 0.08 \, \frac{d}{p}
\]  
(Eq. 6)

and the average crack width \( W_m \) is given by

\[
W_m = S_b \varepsilon_m
\]  
(Eq. 7)

where the average gross strain \( \varepsilon_m \) defined by Beeby is:

\[
\varepsilon_m = \varepsilon_{S2} - \frac{K f_t'}{E_p} \frac{f_{S2,cr}}{f_{S2}}
\]  
(Eq. 8)

where \( K \) is a constant depending on the type of bar and \( f_t' \) is the tensile strength of concrete.
Comparison of Computed and Measured Crack Spacing

It was observed that most of the surface cracks for all the tested specimens coincided approximately with the locations of the transverse reinforcing bars. The average crack spacing between through cracks decreased as more cracks were formed, and the final crack pattern was reached at an average gross strain $\varepsilon_m$ of 0.0011 as shown in Figure 4, for a typical specimen, T2A.

Using Leonhardt's expression (Eq. 1), and Beeby's expression (Eq. 6), the crack spacing were computed and then compared with the measured values from the test.

Figure 5(a) illustrates the comparison between the ratio of Beeby's computed values to the measured values $S_b/S_{exp}$, as a function of bar diameter divided by steel percentage ($d/p$). The comparison suggests an underestimation of the average crack spacing by Beeby, since the average ratio between the computed and measured values is approximately 0.70.

Figure 5(b) clearly indicates that the spacing based on Leonhardt gives values closer to the test results, since the average value of the ratio between predicted and measured values is 1.13, from which it may be concluded that the Leonhardt expression is more valid and provides better accuracy than Beeby's in the prediction of crack spacings in the presence of transverse reinforcement.

A Proposed Modification for Crack Spacing Prediction

In this study an attempt was made to refine the crack spacing expression using Beeby's expression with the addition of the almost no-bond length concept introduced by Leonhardt. The discrepancy between the crack spacing values predicted by Beeby and the measured values were used to re-evaluate the almost no bond length ($L_0$). Based on the experimental results, the proposed expression is as follows:

$$L_{om} = 10 (d - 0.28) \quad (d \text{ in inches}) \quad (Eq. 9)$$

where $L_{om}$ is the modified almost no bond length equivalent to the almost no bond length ($L_0$) proposed by Leonhardt (Eq. 2). The details of the analysis used to derive $L_{om}$ are given in Reference 5.

The above expression assumes that the almost no bond length is a function of the diameter of the steel reinforcement bars and is virtually zero for bars with diameters less than 0.28 inches. Another feature of the new expression is that it is expressed in terms of the section properties, which make it much simpler to use than the original formula (Eq. 2) which is a function of the steel stress level. This simplified expression was based on test results from specimens having a constant transverse bar spacing of 3"centre to centre. However it is believed that it should be general enough to be useful in other cases since it was derived using different parameters, such as concrete cover ($c$), reinforcement ratio ($p$) and concrete thickness ($t$). Thus this expression in turn leads to a new equation for predicting the average crack spacing at the final crack pattern, $S_m$ as follows:
\[ S_m = \frac{1}{2} L_{om} + S_b \]  
(Eq. 10)

where \( S_b \) is equal to the original expression for the average crack spacing given by Beeby's Equation (6). Thus, the final form for the proposed expression for prediction of the average crack spacing in this investigation can be written as follows:

\[ S_m = 5(d-0.28) + 1.33c + 0.08 \frac{d}{p} \]  
(Eq. 11)

Figure 6 shows the ratio between the predicted crack spacing based on Eq. (11) and the measured values. The average value of the predicted crack spacing, based on the new equation, was found to be 97 per cent of the average measured values, which is highly acceptable predictive accuracy.

Comparison Between Computed and Measured Crack Widths

The average crack widths at the final crack pattern for all specimens from the first program were calculated using Leonhardt's expression (Eq. 1). Figure 7 suggests very clearly an overestimation of the average crack width resulting from the use of this expression, since the average value of the ratio between the calculated and experimental values is as large as 2.38.

The crack widths for all the specimens were also calculated at the final crack pattern based on Beeby's expression (Eq. 6) compared with the measured values. Figure 8 indicates that Beeby's expression, (Eq. 6) can be used adequately for predicting the average crack width, since the average ratio between both values is 1.16.

Influence of Transverse Reinforcement on Crack Spacing

Beeby [1, 2], Nawy [8], and McGregor et al. [9] have observed some correlation between the spacing of reinforcement parallel to the cracks and the spacing of the cracks themselves. The second experimental program involved testing of nine reinforced concrete specimens which were identical in terms of reinforced ratio \( p = 0.0147 \), concrete cover \( c = 3/4" \), and concrete thickness \( t = 7" \). The only variation introduced in this experimental program was the spacing between transverse reinforcing bars perpendicular to the direction of the applied load. The three spacings considered in this investigation were 2, 4, and 6 inches.

According to the theoretical predictions, if \( p \), \( c \), and \( t \) were constant, the calculated average crack spacing should have the same magnitude. The calculated values for average crack spacings based on Leonhardt's and Beeby's expressions were found to be 5.3 and 3.4 inches respectively. However, as discussed in the introduction, these expressions were formulated based on testing of reinforced concrete specimens without any transverse reinforcements. Based on the proposed equation from the first experimental program, as presented earlier in this paper, the calculated crack spacing was found to be 4.2 in. The average measured and calculated crack spacings
of the three groups tested in the second experimental program are summarized in table (2).

Based on the measured values in Table (2), it is obvious that none of the above theoretical equations are fully applicable in the case of variable transverse reinforcement spacing. This paper proposes a methodology for prediction of the crack spacing with known transverse reinforcement spacing. The method employs the proposed expression (Eq. 11), which was based on the results of the first experimental program.

For specimens with 4" spacing between the transverse reinforcement bars, it was observed that cracks were located at every bar. This phenomenon was identical and repeated for all three specimens in the same group. Mathematically, this behavior could be expressed as:

\[ S_c^{(th)} = S_R \]  
\[ S_c^{(act)} = S_R \]  

where

- \( S_c^{(th)} \) = theoretical average crack spacing based on Eq. (11)
- \( S_c^{(act)} \) = actual average crack spacing
- \( S_R \) = spacing between transverse bars

For the specimens with 6" spacing between transverse reinforcement, cracks were formed first at every location of the reinforcement bar followed by some cracks in between at the final crack pattern at an average gross strain \( \varepsilon_m \) of 0.0011. This behavior suggests that the spacing between the transverse bars was large enough to allow increase of the induced stress up to the tensile strength of concrete, causing an additional crack to form. Thus if the spacing between transverse bars were more than once but less than twice the predicted spacing, an additional crack would be expected. Mathematically, this could also be formulated as

\[ (n + 1) S_c^{(th)} > S_R > (n) S_c^{(th)} \]  
\[ S_c^{(act)} = \frac{S_R}{n+1} \]  

for \( n+1 \) < \( S_c^{(th)} \) < \( n \) \( S_c^{(th)} \)

Specimens from the third group had reinforcement bars located 2 in. apart, which was less than values of crack spacing predicted by Eq. (11). Cracks were randomly located in these specimens with some cracks at the locations of the transverse bars, and this was observed for all the specimens in this group. The average measured crack spacing was found to be 3.41", which is close to the predicted values based on Eq. (11) and Beeby's expression. Mathematically, this could be presented as:
or \( S_R < S_C(\text{th}) \)  \hspace{1cm} \text{(Eq. 14)}

\[
S_C(\text{act}) = S_C(\text{th})
\]

The crack spacings were computed using this proposed methodology and compared favorably with the measured values as given in Table (3). The final crack pattern for the specimens in the second experimental program are compared in Figure 9.

The techniques for predicting crack spacing in the presence of transverse bars are based on only three cases which were considered in this investigation. The behaviour also fits logically with the theoretical justification of the initiation of crack theories in reinforced concrete members. However, further research work is needed to verify the proposed methodology and to include the variation of many other parameters such as the reinforcement ratio \((p)\), concrete thickness \((t)\), and concrete cover \((C)\).

Summary and Conclusions

The following observations and conclusions are based on the two experimental programs discussed in this study.

1. The spacings of the through cracks in reinforced concrete members are affected primarily by the spacing of the transverse reinforcement steel parallel to the direction of the cracks.

2. The number of through cracks increases as the strain increases. A fully developed pattern of cracks is reached at a strain approximately equal to 0.0011.

3. The new expression proposed for the "almost no bond" length Eq. (9) introduces an equally reliable expression as was proposed by Leonhardt. However, this expression is simpler to use since it is expressed in terms of the section properties instead of the steel stress level. The average value of the predicted crack spacing based on the modified expression, Eq. (11), is 97 per cent of the average crack spacing of the measured values.

4. Leonhardt's prediction (Eq. 4) for the average crack width at the stabilized crack pattern, is highly overestimated, since the ratio between the measured values and those calculated by Leonhardt suggest an average value of 2.38.

5. Beeby's expression, (Eq. 7) can be confidently used for predicting the average crack width at the final crack pattern.

6. A methodology has been presented to predict the crack spacing as influenced by the spacing of the transverse reinforcement. Further research is needed to verify the proposed methodology and to include other parameters believed to affect the crack behaviour of reinforced concrete members.
Table 1. OVERVIEW OF THE VARIABLES CONSIDERED IN THE EXPERIMENTAL PROGRAM

<table>
<thead>
<tr>
<th>CONCRETE COVER (C) in</th>
<th>STEEL RATIO (p)</th>
<th>CONCRETE THICKNESS (t) in</th>
<th>SPECIMEN</th>
<th>X-SECTION AREA in²</th>
<th>REINF. BAR SIZE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.0147</td>
<td>5.0</td>
<td>T1A</td>
<td>60.0</td>
<td>#3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.0</td>
<td>T4A</td>
<td>84.0</td>
<td>10M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.0</td>
<td>T7A</td>
<td>110.0</td>
<td>#4</td>
</tr>
<tr>
<td>0.0207</td>
<td></td>
<td>5.0</td>
<td>T2A</td>
<td>60.0</td>
<td>10M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.0</td>
<td>T5A</td>
<td>77.0</td>
<td>#4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.0</td>
<td>T8A</td>
<td>120.0</td>
<td>15M</td>
</tr>
<tr>
<td>0.0294</td>
<td></td>
<td>5.0</td>
<td>T3A</td>
<td>55.0</td>
<td>#4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.0</td>
<td>T6A</td>
<td>84.0</td>
<td>15M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.0</td>
<td>T9A</td>
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<td>#6</td>
</tr>
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</tr>
<tr>
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<td></td>
<td>7.0</td>
<td>T4B</td>
<td>84.0</td>
<td>10M</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.0</td>
<td>T7B</td>
<td>110.0</td>
<td>#4</td>
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<tr>
<td>0.0207</td>
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<td>5.0</td>
<td>T2B</td>
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<td>10M</td>
</tr>
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<td>T5B</td>
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<td>T8B</td>
<td>120.0</td>
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<tr>
<td>0.75</td>
<td>0.0147</td>
<td>7.0</td>
<td>2A,2B,2C</td>
<td>84</td>
<td>10M</td>
</tr>
</tbody>
</table>
TABLE (2)  CALCULATED AND MEASURED AVERAGE CRACK SPACING FOR THE SECOND EXPERIMENTAL PROGRAM

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Transverse Bar Spacing (in)</th>
<th>Crack Spacing Based On:</th>
<th>Average Measured Crack Spacing (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Leonhardt</td>
<td>Beeby</td>
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<td>5.36</td>
<td>3.4</td>
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<td>4A, 4B, 4C</td>
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<td>5.18</td>
<td>3.4</td>
</tr>
<tr>
<td>6A, 6B, 6C</td>
<td>6</td>
<td>3.31</td>
<td>3.4</td>
</tr>
</tbody>
</table>

TABLE (3)  MEASURED AND CALCULATED AVERAGE CRACK SPACING BASED ON THE PROPOSED TECHNIQUE

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Transverse Bar Spacing (in)</th>
<th>Average Measured Crack Spacing (in)</th>
<th>Average Crack Spacing Based on Proposed Technique</th>
</tr>
</thead>
<tbody>
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<td>2</td>
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<td>4.2</td>
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<tr>
<td>4A, 4B, 4C</td>
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<tr>
<td>6A, 6B, 6C</td>
<td>6</td>
<td>3.5</td>
<td>3.0</td>
</tr>
</tbody>
</table>
Figure (2) Test Set Up and Instrumentation

Figure (1) Reinforcement Details of a Typical Specimen
Figure 5(a) Comparison between measured and computed crack spacing based on Beeby's expression

Figure 5(b) Comparison between measured and computed crack spacing based on Leonhardt's Expression
Figure 6. Comparison between measured and computed crack spacing based on (Eq. 11)

Figure 7. Comparison between measured and computed crack width based on Leonhardt's expression.
Figure 8. Comparison of measured and computed crack widths based on Beeby's Expression.
ACKNOWLEDGEMENTS

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